MULTIPLE QUEUES OF AIRCRAFT UNDER TIME-DEPENDENT CONDITIONS*

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ABSTRACT

A second Toronto airport has been proposed on-and-off for the past decade. The adequacy of the present runway system is studied for the important case of all aircraft operating on a single runway. Air traffic projections were inputs to a time-dependent model with separate queues for landings and departures. The methodology furnishes an intermediate ground between formulas for a steady state which is not likely attained, and a detailed runway simulation.

RÉSUMÉ

Depuis les dix dernières années, on a proposé, à différentes reprises, la construction d'un deuxième aéroport à Toronto. Pour déterminer si le système de pistes d'atterrissage actuel est suffisant, nous étudions le cas important où tous les avions utilisent une seule piste. Pour ce faire des prévisions du trafic aérien sont utilisées dans un modèle non-stationnaire, avec des files d'attente distinctes pour les atterrissages et les décollages. Cette méthodologie constitue une approche intermédiaire entre les formules d'état stationnaire, non susceptible d'être atteinte, et une simulation détaillée d'une piste d'atterrissage.

1. INTRODUCTION AND SUMMARY

Continual increases in delays at a given airport raise the question of whether, by careful analysis of the queues for runway service, congestion can be substantially reduced. Until a few years ago, the waiting lines of aircraft for landing had been modelled as Markov processes possessing steady-state transition probabilities. In fact, however, the arrival and departure rates differ by a factor of 3 between the rush-hour peaks and the early afternoon lull.

Koopman (1972) has treated the formation of queues for aircraft landings as varying significantly with the hour of day. With time-dependent Poisson arrival rates $\lambda(t)$, and assuming the queue evolution to be Markovian, he derived a system of weakly coupled stochastic differential equations for the quantities $P_m(t)$, the probability that at time $t$ there are $m$ aircraft waiting to land. Using for $\lambda(t)$ actual flight statistics from Kennedy and La Guicida airports in New York, the equations were solved numerically.

Koopman treated briefly the case of multiple queues, in which the same runway is used for both arrivals and departures. He derived, but studied only qualitatively, a system of $(M + 1)(N + 1)$ differential equations for $P_{mn}(t)$, the probability that at time $t$ there are $m$ aircraft in the air awaiting landing clearance and $n$ in the ground queue awaiting take-off. ($M$ and $N$ denote the respective maxima allowed in the landing and take-off queues at the airport.)

The present paper studies the multiple queueing problem for Pearson International Airport (Toronto). Although this airport has three runways, that is, two parallel and one perpendicular,

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a strong north-west wind is relatively common. Since regulations forbid use of a given runway direction if the cross-wind component exceeds 15 knots, only the perpendicular runway is available for both take-offs and landings. (For details of runway operations see Newell, 1979.)

The single-runway nature of Koopman's model is thus clearly applicable, and to check its adequacy, empirical rates for Toronto landings and departures were used in the equations determining $P_{mn}(t)$. Numerical solutions were then also obtained for projected future air traffic. The results can delineate policy options and trade-offs for Pearson International Airport and the proposed second airport.

2. EQUATIONS

Let $\lambda(t)$ and $\lambda'(t)$ be the respective probabilities per unit time that an aircraft joins the landing queue or the take-off queue. $\mu(t)$ and $\mu'(t)$ are the respective service rates. As derived in Koopman (1972) the probabilities $P_{mn}(t)$ satisfy for $1 \leq m < M, 1 \leq n < N$:

$$dP_{mn}(t)/dt = -(\lambda + \lambda' + \mu + \mu') P_{mn}(t) + \lambda P_{m-1,n}(t) + \lambda' P_{m,n-1}(t) + \mu P_{m+1,n}(t) + \mu' P_{m,n+1}(t).$$

The system (1), an infinite set of differential equations, was truncated by assuming $P_{mn}(t) = 0$ for $m > M$ and all $n$; or for $n > N$ for all $m$. Numerical values for $M$ and $N$ are discussed in section 3.3.

Besides eliminating certain terms from the equation when the queue sizes are maximum, a corresponding modification occurs when $m = 0$ or $n = 0$. Here, the terms involving $(m - 1)$ or $(n - 1)$ must be excluded, as must also the terms $\mu P_{m,n}$ and $\mu' P_{m,n}$.

Equation (1) plus the preceding alterations at boundary values of $m$ or $n$, represents a set of $(M + 1)(N + 1)$ first-order differential equations whose solutions $P_{mn}(t)$ give the probabilities of various queue sizes at each time of day. Initial conditions are obtained by noting that Pearson International Airport has a curfew between midnight and 7 a.m., during which times there are very few movements of aircraft. The initial values are therefore $P_{mm}(6.8) = 1$, and $P_{nn}(6.8) = 0$ if $m \neq 0$ or $n \neq 0$.

3. MODEL VALIDATION

Let us first consider the question of overall validity. This concerns the relationship between outputs, those of our model and those of the real system. Overall face validity was established by conducting an informal "Turing Test." The model was run under varying experimental conditions, using arrival and departure rates from several different years, and the results were discussed with knowledgeable practitioners from Transport Canada and Air Canada. These individuals felt the calculated queue sizes and time dependence were consistent with records of Transport Canada and with the actual queues and delays seen by an on-site observer over a number of successive days.

The empirical data were not detailed enough or available in sufficient quantity to conduct meaningful goodness-of-fit tests on the computed time-dependent distributions of delay or numbers in queue. Although a Turing Test does not completely validate a model, satisfaction of such a test is important and necessary in practice if the model is to be used as an aid in decision-making.
Fortunately, we are able to say more concerning validation of the input parameters and distributions. The set of equations (1) of course assumes that service times and interarrival times are exponentially distributed. Sections 3.1 and 3.2 furnish arguments in support of this choice, while section 3.3 concerns the maximum queue sizes $M,N$.

### 3.1 Interarrival times

Hengsbach and Odoni (1975) point out that the assumption of Poisson arrivals for airport “demand,” that is, for landings or for take-offs, is consistent with actual observations at major airports and has been extensively used in the transportation literature. This of course is equivalent to an exponential distribution of interarrival times.

Stanford, Pagurek and Woodside have studied the optimal prediction of queue lengths and delays in the $GI/M/1$ queue (1983) and in $GI/M/c$ queues (1984). Their procedures may be used, for example, to relate the number in a queuing system to a future time, to the numbers at appropriate times in the recent past. Application to our airport example of these methods, of which need little or no modification to handle non-stationary processes, would be a way to validate use of (1) when the interarrival distribution may not be exponential.

### 3.2 Service times

Aircraft service times, whether for take-offs or landings, are not likely exponential. Several points may be advanced, however, in favour of our model’s robustness. For landings Koopman (1972) has compared the queue lengths with a constant service time with those attained under exponential service with the same mean. In each case the empirical arrival rate $\lambda(t)$ was used, and the average queue length $m(t)$ was plotted as a function of time of day. Koopman found only a small band between the $m(t)$, for each case, the respective curves twice crossing each other during the day’s operation. The band between the two curves was considerably less than the standard deviation of either, since the true service distribution has a “randomness” somewhere between that of the exponential and constant distributions, one may employ the exponential service times required by the model (1). A similar argument was made by Bookbinder and Martell (1979) in the context of a different application.

### 3.3 Maximum queue sizes

The final input parameters needed for our model are the maximum queue sizes $M$ and $N$. These should be chosen larger than the waiting lines likely to be encountered with the given arrival and service rates, so that only with low probability would an arriving aircraft be rejected by the system.

We employed $M = N = 6$ for the 1990 forecasts and $M,N = 7$ or 8 for 1995. Each guess for the pair $(M,N)$ was checked by comparison of the results for $P_{md}(t)$ obtained for $(M,N)$ with those found for the case $(M + 1, N + 1)$. In each of the respective instances the magnitudes of $P_{M+1,0}$ and $P_{N+1,N}$ were found to be of order $10^{-3}$, even in the peak periods, while the probabilities $P_{md}(t)$ for smaller queue sizes were virtually unchanged from before, and so the original choices $(M,N)$ were retained. We also did several runs employing $(M + 2)$ and $(N + 2)$, with similar results. For further discussion of the numerical procedures see Bookbinder (1976), Bookbinder and Luthra (1974), and Bookbinder and Martell (1979).
4. Related Work

Before describing our results, let us briefly discuss some related research. Hengsbach and Odoni (1975) used an approach similar to that of Koopman (1972) to estimate the time-dependent marginal cost due to aircraft delay. Roth (1979) has extended the work of Hengsbach and Odoni (1975) and Koopman (1972) to include service rates dependent upon the preceding operation (landing or departure) and has studied various priorities between landings and take-offs. Nair (1976) has also compared several priority schemes for the case of two queues attended by a single server. We remark that Brandwajn (1979) has considered a general two-dimensional birth-and-death process, for the case where there exists a steady state.

Kolesar et al. (1975) employed Koopman's equations to estimate the transient delays of urban police cars in responding to alarms. Bookbinder and Martell (1979) have used the approach of Bookbinder (1976) and Bookbinder and Luthra (1974) to allocate helicopters for the initial strike on forest fires.

Other methods are of course available in addition to solution of the time-dependent birth-and-death equations. The main alternatives are simulation (Atack, 1978; Fishman and Kao, 1977) and various approximation schemes (Collings and Stoneman, 1976; Moore, 1975; Rider, 1976; Rothkopf and Oren, 1979). Atack (1978) has developed a detailed runway simulation model for an airport project in Sydney, Australia. Fishman and Kao (1977) have clarified some of the difficulties that arise in queueing simulations when the parameters are time dependent. Collings and Stoneman (1976) obtained an analytic solution for the queue-length distribution in the limiting case $M(t)/M(t)/\infty$. Rothkopf and Oren (1979) solved the nonstationary problem for a finite number of servers by making a "closure approximation," an assumption of the functional relationship between some of the system variables, to reduce an infinite set of equations to a finite set. Rider (1976) has approximated the mean of a non-stationary $M/M/1$ queue by a type of closure assumption, while Moore (1975) has used an embedded Markov chain approach to study this same queueing system.

5. Input Data

The system of equations contains as coefficients the arrival and departure rates $\lambda(t)$ and $\lambda'(t)$, and the service rates $\mu(t)$ and $\mu'(t)$. The 1990 and 1995 forecasts of $\lambda(t)$ and $\lambda'(t)$, presented in table 1, represent a consensus between two independent sources, one from the Canadian government and one from an airline.

The service rates $\mu$ and $\mu'$ for commercial aircraft are expected to be longer than for general aviation. Depending upon the proportions present of each aircraft type, the rates $\mu$ and $\mu'$ will thus vary with time of day. The weighted average for landings, $\mu_{\text{eff}}(t)$, was handled as

$$\frac{1}{\mu_{\text{eff}}} = \left[ \lambda_c(1/\mu_c) + \lambda_s(1/\mu_s) \right] / (\lambda_c + \lambda_s),$$

where $\mu_c = 45/\text{hr}$ is a conservative estimate for the service rate appropriate to commercial landings only, while $\mu_s = 65/\text{hr}$ applies to general aviation only. $\lambda_c(t)$, $\lambda_s(t)$ of course refer to the probabilities per unit time of landings of the respective aircraft type, at each time of day $t$. Similarly, the effective service rate $\mu'_{\text{eff}}$ for take-offs is
TABLE I
ARRIVAL AND DEPARTURE RATES: MEDIUM AND LONG-TERM FORECASTS (COMMERCIAL AIRCRAFT)

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\[
1/\mu'_{\text{eff}} = \left[ \lambda'(1/\mu'_{\text{c}}) + \lambda'_g (1/\mu'_g) \right] (\lambda'_c + \lambda'_g),
\]  

where the primes denote take-offs and \( \mu'_{\text{c}} = 50/\text{hr} \), \( \mu'_g = 70/\text{hr} \), choices that again are conservative.

The rates \( \lambda \) and \( \lambda' \) and the effective service times of equations (2) and (3) form the "raw" input data to the system (1). Use of the piecewise constant \( \lambda(t) \), \( \lambda'(t) \) indicated by table 1, however, would yield probabilities \( P_{\text{out}}(t) \) whose time derivatives have occasional jump-discontinuities. This is counter-intuitive but results because the rates forecast for the next decade had no finer grid than 1 hour. The "physical" arrival processes \( \lambda(t) \) or \( \lambda'(t) \) do not have such discontinuities on the hour. Rather, they should vary smoothly from one hourly arrival rate to the next, in a pattern more like that of a set of upward or downward sloping ramps interspersed between the horizontal plateaus. This is in fact the time behaviour that one observes empirically.

Accordingly, we employed as the input rates, a 3-point moving average over an interval of 0.2 hrs on either side of a given time \( t \). These moving averages for \( \lambda(t) \) and \( \lambda'(t) \) were used in (1) and in calculating \( \mu'_{\text{eff}} \) and \( \mu'_g \) via equations (2) and (3). The differential equations were solved using a double precision, Runge-Kutta approach based on the fourth-order formulas of Fehlberg (1970). For further details of the numerical procedures, see Bookbinder (1976), Bookbinder and Luthra (1974), and Bookbinder and Martell (1979).

Another remark may be made concerning continuity effects at the hourly boundaries. The airlines' current scheduling practices are that many flights are scheduled to take off on the hour, for example, at 6 p.m. \( (t = 18.0 \text{ hrs}) \), with none or few scheduled at \( t = 17.8 \) or \( t = 18.2 \).
In practice, competing air carriers take turns in planning to depart Toronto just on time, slightly late and a bit later. This is the managerial reason for employing the moving-average arrival rates $\lambda(t), \lambda'(t)$, use of which makes sense mathematically as well.

6. Queuing Discipline

Equation (1) does not follow the evolution in time of a particular aircraft from "arrival," to clearance to land, to completion of service. As opposed to a detailed runway simulation (see Atack, 1978), our model can thus not be expected to reflect the priorities between landings and take-offs assigned dynamically by a controller. An air traffic controller may shift priorities in response to unanticipated events, but generally these priorities are as follows (Newell, 1979). In peak periods operations (on a single runway) will alternate between landings and take-offs to increase throughput; in off-peak periods landings usually have priority (non-pre-emptive) over departures.

The implicit assumption of equation (1) is that equal priorities are assigned to landings and take-offs, and that service is first come, first served within each queue. In off-peak periods, that is, those in which runway capacity is not a factor, the queuing discipline should not be an issue; either way, aircraft will be served and queues will not build up. In the peak periods, with the controller alternating between the landing and take-off queues, our model's assumption of equal priorities should be a good approximation.

7. Results and Discussion

Once solutions for the probabilities $P_m(t)$ have been obtained, quantities such as $\overline{m}$ and $\overline{n}$ (the expected sizes of the landing and the take-off queues) can then be calculated. These mean numbers in the queues were not found to be very informative since, as is often the case for queueing systems, the standard deviation of each queue length was greater than its mean. We have therefore concentrated on the 0.95 fractiles $M_f$ and $N_f$ obtained by studying the respective marginal distributions of the landing and departure queues. The knowledge that 95% of the time there are $M_f$ or fewer aircraft waiting to land, conveys more to an airport policy planner than do statements based upon means and standard deviations.

Projections for 1990 and 1995 of the sum $(M_f + N_f)$ by time of day, typical of what can be obtained from our procedure, appear in figures 1 and 2. The arrival rates $\lambda$ and $\lambda'$ for the upper curves included general aviation aircraft in addition to the commercial traffic of table 1. The effect of general aviation is a broadening of the time-width of the peak but little increase in the maximum total sizes $(M_f + N_f)$. The additional queuing due to general aviation occurs away from (but adjacent to) the (former) peak hour. (We recall that the model includes separate queues for landings and departures. The combined total size $(M_f + N_f)$ is used only as a convenient summary measure of delay.)

In each figure the difference in queue lengths between (general and commercial) and only commercial aircraft is at most two planes, about the same as the difference in queue sizes between 1990 and 1995 for solely commercial aircraft. This has indicated to policy planners that, other things being equal, a second major airport can be delayed for at least five to ten years, by shifting all general aviation to a nearby municipal airport (which can handle only
Fig. 1. 1990 forecasts of 0.95 fractiles for combined lengths of each queue

Fig. 2. 1995 forecasts of 0.95 fractiles for combined lengths of each queue
small planes). Such a shift is recommended for the future but need not be immediately enforced, in view of the broadened time-width mentioned above.

In closing, we wish to compare our method with the other common approaches, that is, use of steady state formulas or a computer simulation of runway operations (Atack, 1978). Because the arrival rates were not constant but varied significantly throughout the period of interest, a steady-state solution may not be attained and may be meaningful only as an upper bound, that is, if the steady-state probabilities were calculated with respect to the daily maxima of the arrival rates \( \lambda, \lambda' \). On the other hand, many researchers tend to feel that if a problem cannot be solved in "closed form," then simulation is the only option. (In fact, of course, there are also the approximation schemes (see section 4).

Numerical solution of the time-dependent queueing equations is an intermediate course. It will produce directly what a simulation model can achieve only after a considerable number of repetitive runs and even then only in the form of a confidence interval.

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