A DRP-Approach to the Management of Rail Car Inventories

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ABSTRACT

Distribution Requirements Planning (DRP) is a relatively new approach to inventory management for goods held at multiple locations. It yields a plan of inventories, for stock at each location and for the quantities to be shipped between successive levels or echelons in the distribution system. DRP implementations often concern inventories of retail goods, but the present paper applies DRP to the management of rail car inventories. A DRP plan covers movements of both full and empty cars over a planning horizon of T days. The initial DRP plan is complete only for the first few days, but that plan is extended by using a transshipment model whose solution gives the daily flows of empty rail cars between each pair of nodes. This model minimizes the total cost of car movements subject to dynamic supply and demand constraints, in which empty cars available and required at each location are based upon actual or anticipated full-car shipments of goods. The quality of the railway's DRP plan thus depends upon good communication with major shippers concerning their forecasts of demand for rail cars. Numerical examples are presented, and generalizations of our model are suggested.

I. Introduction

Railways are facing increased competition from other modes, particularly trucking and inter-modal ("piggy-back") services. The pressures of deregulation have made it difficult to maintain traffic volumes, or freight rates, or both.

One avenue for a railway to become more competitive is by optimizing the movement of rail cars. Johnson and Kovitch [12] discuss the need to control inventories of empty cars in order to buffer supply and demand at loading points. If the causes of inventory fluctuation can be identified, a system may effectively control them. More recently, Tyworth [22] analyzed the utilization by shippers of private freight cars. He found that when the owner carefully...
monitored detention of cars, there was considerable improvement in the speed at which those cars were released by the shipper. Performance of the rail carrier was a less significant factor.

In the present paper, we study the management of rail car inventories using Distribution Requirements Planning (DRP) [17]. This technique has been successful in the control of stocks of retail goods held at multiple locations. For completeness, the essentials of DRP will be given below. We then introduce a transshipment model which, when used in conjunction with DRP, enables development of a minimum-cost inventory plan for the distribution of rail cars. This multi-period DRP plan is up-dated over time as cars are dispatched and returned, and as further forecasts of planned rail shipments are obtained from major customers.

Numerical examples will be given of the use of this model in managing a given fleet of cars. We also show how a variation of our transshipment model can be employed to obtain the "optimal" fleet size, the minimum supply of rail cars which can meet a given inventory/distribution plan. We begin with a summary of the literature.

II. Review of Literature

Economic and public-policy considerations are intertwined in most operational aspects of Canadian transportation, and the rail mode is no exception. An excellent introduction to rail freight transportation in Canada has been given by Clark and Piper [9]. Some public-policy issues concerning rail freight are covered in [19] regarding the Province of Ontario and in [14] for the United States environment. Economic aspects of rail car distribution have been discussed in [3] and [11].

In this paper, however, we will concentrate solely on operational issues, one of which is the routing of shipments. Bronzini and Sherman [7] find each shipment's minimum-cost path through a rail network, taking into account that a portion of the route may be on the line of a competing railway. Models for route selection and other analytical models in rail transportation are surveyed by Assad [2].

The major operational issue which concerns us is rail car distribution. Several approaches [1, 8, 10, 18] to this problem have been based on single-period linear programming (LP) models. Allman [1] treats car distribution as a case of the Assignment Problem, and tries to minimize "per diem" rental rates, i.e., the rate paid by one railway to another for use of its cars. Although this problem involves both distribution (what to do with the cars on hand) as well as inventory (how many should be on hand), only the distribution aspect is discussed. Charnes and Miller [8] present a linear programming model to minimize costs, while the LP model of Misra [18] minimizes empty car-hours.

Feeney [10] treats car distribution as a single-period, probabilistic LP in which supply and demand are random variables.

Multi-Period Models of Car Distribution

The preceding treatments [1, 8, 10, 18] have outlined the general issues, but by not addressing the recurrent nature of this problem, little assistance is furnished to a railway that wishes to plan several periods ahead. A multi-period model of car distribution is required. Relevant publications include [6, 15, 20] which use LP, and [13, 16, 24, 25] which are transshipment models.

Linear Programming Models

Bookbinder and Sethi [6] discuss the "dynamic transportation problem," in this case a multi-period deterministic LP to minimize the sum of shipping costs and inventory holding costs. There are some similarities, but also an important distinction, between the car-distribution problem and that type of transportation/inventory model. An obvious aspect of this problem is that rail cars are re-usable. To apply that model to the distribution of rail cars would require assuming that once a car has been allocated, it is out of the system and does not re-enter when it is emptied.

Konya [15] uses a multi-period LP to minimize the costs of empty-car travel plus shortages. Travel times for both loaded and empty cars are probabilistic. Again, inventory is not considered re-usable.

Ratcliffe et al [20] employ a hybrid technique: a probabilistic simulation to generate supplies and demands, and then a deterministic LP to minimize travel time of empty cars. Inventory is considered re-usable, with the supply and demand at each time period determined by the simulation. This approach would appear useful in generating probability distributions for rail-car utilization and service to shippers.

Transshipment Models

White [24] and White and Bomberault [25] have studied the transshipment problem, pertaining respectively to the distribution of empty containers and to empty freight-car allocation. This approach is interesting in that, rather than considering a node as a location only, each node designates a location at a particular time.

Jordan and Turnquist [13] have developed a multi-period transshipment model, with new orders and supply and travel times given as probabilistic. Its objective is to maximize profit. The model makes a decision in each period of which cars to move. It does not allow for moving a car in two legs; the car would be moved directly to the required location.

In a related application, Love [16] deals with a two-location vehicle rental
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between these two dates, where that lead-time is the total interval between release of an order from this local warehouse to the central warehouse, until those goods are actually available at the former location.

In summary, DRP involves a plan of inventories, for stock at a given location and for quantities shipped between successive levels or echelons in the distribution system. At any stock-keeping location, the multi-period plan may be summarized in a “tableau” in which columns represent time periods, and where there are rows for Gross and Net Requirements, Quantity on Hand, Planned Order Releases and Scheduled Receipts. The appropriate “planning horizon,” the number of columns in each DRP tableau, is the longest cumulative system lead-time from the lowest echelon (retailer) up through a replenishment at the central warehouse. Further details of DRP are contained in[4, 17, 23].

The present paper concerns not the inventory of goods but of rail cars. Naturally, to describe the planned dispatches and receipts of cars, the preceding DRP terminology will be adapted to the case of a railway, and additional rows included for clarity. (A tableau is presented later in Table 2).

A DRP plan for a railway would note that demand for cars at each local car-distribution centre is dependent on the demand for rail cars by its customers, the shippers of goods. We would argue that, say, 80 percent of the railway’s total demand for cars comes from 20 percent of their customers. These are the major shippers, the ones which often have a systematic plan for goods distribution. Admittedly, such a plan is revised over time because of certain random factors in each company's business or market. Nevertheless, advanced knowledge of the initial plan will provide an almost deterministic forecast of demand for cars in the first few days, and furnish a far better basis to plan several days beyond that, than would standard statistical methods.

IV. Proposed Transshipment Model

In this section we develop our Transshipment Model, referred to as TSM. We relate the DRP plan of the railway to the solution of this model and to the planned shipments for each location over the DRP planning horizon.

It is assumed that the railway can forecast $y_{ik}$, the demand for rail cars in which full car-load shipments of goods will occur on day $k$ between origin $i$ and destination $j$. As above, such a forecast is assumed to begin with advance communication from major shippers of their desires to send full cars between
the number of full cars moved on day k from i to j on the first leg of a transshipment. For \( v_{ijk} \), this full-car movement from i to j on day k represents the second leg of a transshipment. We found that use of 3 distinct variables for full-car shipments was both helpful in interpreting the model outputs and necessary to avoid double-counting some movements of cars.

The costs are respectively \( a_{ijk} \) to move an empty car and \( b_{ijk} \) to move a full car. One naturally expects \( a_{ijk} < b_{ijk} \) for all i, j, k. In our numerical examples, we have assumed there is no time-dependence in these cost parameters.

The objective function \( Z \) is minimized subject to the following constraints, in which \( S_{So} \) is the initial supply (on day 0) of rail cars at location i.

**Supply Constraints**

Supply constraints are written as:

\[
\begin{align*}
(2) \quad \sum_{j \neq i} (x_{ijk} + y_{ijk} + v_{ijk}) & \leq S_{So} + \\
& \text{for all } i, j, k \geq 1
\end{align*}
\]

With the initial supply of cars from DRP and its update as above, plus the Gross requirements for cars, TSM is now complete. Solution of this model yields information such as the shipments planned for a given location during the planning horizon, and all movements in the network for a given day. Examples of these and other model outputs, as well as sensitivity analyses of interest, are given in the next section. A statement of TSM now follows.

**The Objective Function**

The objective function is written as follows:

\[
\begin{align*}
(1) \quad \text{Min } Z = \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=0}^{T} & [a_{ijk} x_{ijk} + b_{ijk} (v_{ijk} + y_{ijk})] \\
& + b_{ijk} (v_{ijk} + y_{ijk} + v_{ijk})
\end{align*}
\]

The subscripts i, j, k respectively indicate origin, destination and day. Thus, \( x_{ijk} \) is the number of empty cars which leave location i on day k, enroute to location j. (We assume throughout this paper that, for simplicity, exactly 1 day is required to traverse an arc between any pair of the N nodes in the network). The variables \( y_{ijk}, u_{ijk} \) and \( v_{ijk} \) all refer to full-car movements on day k within the T-period planning horizon. \( y_{ijk} \) is the number of full cars moved directly from i to j, while the u- and v-variables denote transshipments. \( u_{ijk} \) is
corrections to $S_o$ on the right-hand side. A full car which enters $i$ on the first leg of a transshipment contains goods required by a consignee at some other location $p$. Once those goods have been delivered, the now-empty car contributes to the supply of cars at $p$.

Eq. (2) holds for $2 \leq k \leq T$. For $k=1$, the terms in $y_{nim}$ and $v_{nim}$ are to be taken as zero. Similarly, for $k=0$, the start of the planning horizon, only $S_o$ is to be included on the right-hand side.

**Demand Constraints**

Demand constraints consist of the following:

$$
S_o + \sum_{n=1}^{N} \sum_{m=0}^{k-1} x_{nim} + \sum_{m=0}^{k} (y_{nim} + v_{nim}) \\
\geq D_k
$$

Eq. (3) is valid for $k \geq 2$, with modifications similar to those above for $k = 1$ or 0. As the time $k$ progresses, the demand $D_k$ for rail cars at location $i$ will be filled by cars which were at other nodes in earlier periods, and indeed may be returning to site $i$. This re-use of rail cars, as mentioned, makes Eqs. (2) and (3) look much more imposing than the supply and demand constraints in the standard transportation problem.

**Discussion**

The objective function was expressed in the preceding form for three reasons:

(i) It expresses the cost incurred from the total of full and empty movements;

(ii) Some LP software (including the package we employed) does not allow a variable to appear in a constraint unless it also appears in the objective function;

(iii) The expression (1) suggests a possible generalization as a stochastic programming model, in which the objective is to minimize expected cost, where the full-car movements ($y$, $u$, $v$) are statistical forecasts subject to uncertainty, and the constraints are each to be satisfied with a certain probability.

Nevertheless, in the analysis which follows, the desired full-car movements are assumed known by our previous argument. (As in the standard transshipment problem, any full-car movements that are more appropriately transshipped can easily be determined from distance and cost relationships in the network.) Thus, it is the empty movements which comprise the decision variables of TSM.

**V. An Example**

As an example of the use of our model, consider $N=3$ locations and the summary in Table 1 of full-car shipments to be carried by the railway during a 5-day horizon. Suppose the DRP plan (Table 2) includes 3 days of demand and 5 days of supply. (The first two days of supply concern cars whose availability results from activities in a previous DRP plan, e.g., cars released after being unloaded). The Gross Requirements in the DRP plan for each location $i$ specifies the demand there for empty cars. Some of these requirements will be covered by supplies currently available or ready to be released from consignees. However, when the tableau shows that Net Requirements at $i$ are positive, additional cars must be obtained.

Where will they come from? Solution of the transshipment model gives the minimum-cost plan of empty-car movements which satisfy the Net Requirements. Those are the cars scheduled for transport to $i$, and planned to be received there an appropriate lead time later. Thus, empty cars are moved to location $i$, if and only if the Net Requirements there are non-zero. Naturally, a shipper at $i$ will then load each car with goods intended for delivery elsewhere.

Tables 3 and 4 summarize the optimal car movements pertaining to the DRP plan of Table 2. Note, in Table 2, there are positive Net Requirements at location 3 on day 3 (and on day 4; all other Net Requirements are zero). The empty movements needed to satisfy these requirements is the only difference between Table 4 and the day-3 entries of Table 1. A complete daily operating plan for the railway would be like Table 4. It is obtained in our approach by combining DRP and the transshipment model with the customers' plans of full-car shipments given in Table 1.

**VI. The "Optimal" Supply of Rail Cars**

In the preceding statement of our problem, the supply of rail cars was given. There were $S_o$ cars at each location $i$ in constraints (2) and (3). By minimizing the objective function (1), subject to those constraints, the railway will have done as well as it could under the circumstances.

However, there is potentially a great difference between "optimizing a given system" and "designing an optimal system." Zeleny [26] has used the term *de Novo Programming* to refer to the latter activity. In the context of a linear
TABLE 1
FULL-CAR SHIPMENTS TO BE CARRIED BY THE RAILWAY DURING THE PLANNING HORIZON

<table>
<thead>
<tr>
<th>FROM</th>
<th>Day 0</th>
<th>Day 1</th>
<th>Day 2</th>
<th>Day 3</th>
<th>Day 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2 3</td>
<td>1 2 3</td>
<td>1 2 3</td>
<td>1 2 3</td>
<td>1 2 3</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>10</td>
<td>25</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>14</td>
<td>35**</td>
<td>0</td>
<td>5</td>
</tr>
</tbody>
</table>

* First leg of transshipment; do not unload yet.
** Second leg of a transshipment; unload at destination.

TABLE 2
THE DRP PLANS FOR LOCATIONS (1, 2, 3)

<table>
<thead>
<tr>
<th>Gross Requirements</th>
<th>Day 0</th>
<th>Day 1</th>
<th>Day 2</th>
<th>Day 3</th>
<th>Day 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(35,20,15)</td>
<td>(35,35,14)</td>
<td>(15,26,5)</td>
<td>(42,51,40)</td>
<td>(18,23,20)</td>
</tr>
<tr>
<td>Full cars sent out, Direct</td>
<td>(10,20,15)</td>
<td>(35,35,14)</td>
<td>(15,15,6)</td>
<td>(42,46,40)</td>
<td>(18,23,20)</td>
</tr>
<tr>
<td>Full cars transshipped out</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>First leg</td>
<td>(35,0,0)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Second leg</td>
<td>(25,0,0)</td>
<td>(0,0,25)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Empty cars sent out 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Full cars arriving, Direct</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(A)</td>
<td>(0,0,35)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(B)</td>
<td>(15,35,10)</td>
<td>(39,60,20)</td>
<td>(15,15,15)</td>
<td>(39,43,46)</td>
<td></td>
</tr>
<tr>
<td>Empty cars arriving, Transshipped</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Direct</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Available to Promise,</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(A)</td>
<td>(41,35,18)</td>
<td>(4,0,4)</td>
<td>(6,9,9)</td>
<td>(3,18,0)</td>
<td></td>
</tr>
</tbody>
</table>

1 For empty cars, either to be filled by a shipper at this location, or sent out empty to be filled at another location.
2 No need to distinguish between transshipments and direct movements, since each depletes the supply of empty cars.
3 Includes both direct movements and second leg of transshipment.

NOTE: The cells in this table are to be read as follows. An entry of the form \((w_1, w_2, w_3)\) in a row on a certain day denotes respective values of \(w_1\) at location \(i\) for that day. Cells that are blank are to be interpreted as \((0, 0, 0)\).

TABLE 3
SHIPMENTS IN AND OUT OF LOCATION 3

<table>
<thead>
<tr>
<th>Day 0</th>
<th>Day 1</th>
<th>Day 2</th>
<th>Day 3</th>
<th>Day 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>In</td>
<td>Out</td>
<td>In</td>
<td>Out</td>
<td>In</td>
</tr>
<tr>
<td>Full 1</td>
<td>5</td>
<td>35**</td>
<td>14</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>35**</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>empty 1</td>
<td>2</td>
<td>11</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

When "in," full i refers to full cars coming in from location i. When "out," full i refers to full cars going to location i. Similarly for empty cars.

When shipments 'out' are listed on the day they are dispatched.

Shipments 'in' are listed on the day that the car is "Available to Promise." [Full cars are available 2 days after dispatch, empty cars one day]. However, full cars sent on first leg of transshipment are shown on day of arrival, since they are not unloaded.

* First leg of transshipment.
** Second leg of transshipment.

TABLE 4
PLANNED SYSTEM MOVEMENTS, DAY 3

<table>
<thead>
<tr>
<th>From/To</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>28</td>
<td>16</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>26</td>
<td>16</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
<td></td>
<td>30; 11*</td>
</tr>
<tr>
<td>3</td>
<td>23</td>
<td>17</td>
<td></td>
</tr>
</tbody>
</table>

* Denotes empty movement
(entries without asterisk signify full-car movements)

programming model for profit maximization. Zeleny shows how the optimal system may be obtained by treating what had been the right-hand-side of a resource-constraint as a decision variable. This is then included in the objective function, with a coefficient representing the acquisition cost per unit of

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this resource. The optimal solution of the new linear program yields, in addition to (possibly modified) values of the original decision variables, the optimal amount of resource which should be acquired. This is the “optimal system.”

In the context of the rail-car distribution problem, Zeleny’s approach suggests that we treat the initial supply $S_0$ of cars at location $i$ as a decision variable $B_i$. This requires only minor modification of our formulation. In the constraints (2) and (3), $S_0$ would be replaced by $B_i$. An additional term $\sum B_i$ is added to the objective function. The parameter $c_i$ may be thought of as the cost to acquire one rail car. Further interpretation of $c_i$ is given below.

Table 5 shows that the optimal system in our case contains 189 total cars. We remark that Tables 2 through 4 were based on this optimal system. That can be seen in Table 2, for example, by the entry on day $d$ of (0,0,0) cars Available to Promise.

Table 5 also indicates sensitivity of the model (relative costs) when the total supply of rail cars is varied. Several points should be kept in mind. The relative costs naturally depend to some extent on our chosen numerical values $a_{ki}$, $c_i$, etc. As well, the empty car movements are clearly a consequence of the particular DRP plan (Table 2) and the given requirements of shippers (Table 1).

Indeed, Table 5 may be viewed as the most favorable case, in which when each extra car is added to the system, it is placed so as to eliminate an empty movement. It would be clearly less cost-effective to put an extra car at a location whose Net Requirements were zero. Relative cost increases larger than those of Table 5 would therefore be expected in practice if additional cars were dispersed around the system, including nodes with an excess car supply.

Even a one percent cost saving should not be taken lightly, however. Full-car movements are associated with revenues, while empty movements are pure cost. This one percent saving would “go right to the bottom-line,” and may be quite significant in these times of fragile operating ratios.

Although the parameter $c_i$ may be thought of as the cost “to acquire one rail car,” this interpretation should not be taken literally:

(i) Our objective function (1) deals with costs (rather than with profit as in Zeleny’s case [26]). We have not considered the revenues necessary to offset these acquisition costs.

(ii) As already noted, our resources (the rail cars) are re-usable.

Thus, for de Novo programming with a cost objective and in the context of rail car inventories, $c_i$ is more of a device to aid in determining the “optimal system,” the minimum number of cars necessary to execute the DRP plan. The parameter $c_i$ also permits us to compare the cost of systems with a greater

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**Table 5**

<table>
<thead>
<tr>
<th>Total No. of Cars</th>
<th>No. Empty Cars Moved</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>188</td>
<td>16</td>
<td>1.00</td>
</tr>
<tr>
<td>194</td>
<td>11</td>
<td>1.01</td>
</tr>
<tr>
<td>199</td>
<td>6</td>
<td>1.02</td>
</tr>
<tr>
<td>204</td>
<td>1</td>
<td>1.03</td>
</tr>
<tr>
<td>205</td>
<td>0</td>
<td>1.04</td>
</tr>
</tbody>
</table>

1. This is $\sum B_i$. We took $c_i = c$ for all $i$.
2. Sum over the planning horizon.
3. Relative to Cost of the “Optimal System.”
4. Optimal No. of Cars as selected by model.

number of cars to the cost of the optimal system (Table 5).

It may be argued that a railway cannot operate with so little slack that it has precisely the number of cars needed to execute the DRP plan, and no more. The number of extra cars beyond those of the optimal system could at least be deemed “reasonable” or not, by comparison with results such as those of Table 5. One reason that more than a few additional cars might not be deemed reasonable is that the rail fleet will be managed using DRP. If, as mentioned above, the railway receives good communication concerning intended shipments by its major customers, much uncertainty in the demand for cars will have been reduced.

An interesting policy for the railway would be to charge a reduced rate per car movement to those major shippers who furnish advance information concerning distribution plans which are fairly firm. This rate would include an appropriate profit component; the number of cars would reflect the optimally-designed system. Then, demands for additional car movements, ones not covered in the “firm” portion of the railway’s DRP plan, would be charged at a higher rate. As well, a longer lead time would be quoted for delivery of an empty car to be loaded. The analogous policy for a trucking company was studied by Bookbinder and Lynn [5].
VII. Conclusions and Extensions to our Model

In this paper, we have shown how Distribution Requirements Planning [17] can be applied to the management of a fleet of rail cars. Our approach involves a transshipment model for the minimum-cost pattern of empty-car movements, plus a DRP plan containing these and the full-car requirements and shipments. The full-car movements anticipated during the planning horizon, hence included in the DRP plan, appear in the transshipment model via dynamic supply and demand constraints.

Naturally, as time progresses, the major shippers will know more precisely their needs for rail cars. When the railway is advised of those changes to their full-car requirements, the DRP plan is up-dated, which means the transshipment model must be re-run to obtain the revised plan of empty-car movements. As in the case of retail inventories, DRP should provide a good tool to keep up-to-date a multi-period plan of the inventories of rail cars.

It is because revisions to the plan are entirely expected, that there would be no point in practice of solving an infinite-horizon problem. Rather, one solves a T-period problem (e.g. T=10 to 14 days), but implements only the decisions of the first few periods, say up to t=τ. In period (τ + 1), one solves a new T-period problem containing revised data for those periods overlapping the first problem. Further discussion of “rolling-horizon” planning is contained in [4, 23].

The DRP plan and transshipment model were illustrated for an example with three locations. It is easy to include more nodes; each location will have its own DRP tableau. As additional supply points are considered, eventually an extra level or echelon will be appropriate for the car management system. This corresponds to regional car-distribution centres besides the present local centres (i = 1, 2, 3) of car supply. There will thus be another level of DRP dependent-demand and hence an additional lead-time offset which must be considered. This is standard fare in DRP [4, 17, 23].

Of course, a railway has several types of cars. The variables in our model would then be of the form x_{ijkq}, for empty movements of a car of type q. The objective function Z would then have cost coefficients c_{ijkq}, etc., and would now include a sum over q. There would also be q times as many supply and demand constraints, a very straightforward extension. Note the individual varieties of cars play the same role here as do different “products” in a DRP plan for inventories of consumer goods. Thus, for every location, there will be a separate DRP plan for each car type.

We have assumed that, whether to traverse an arc in the network or to unload a full car and make it ready as an empty, exactly 1 day is required. Suppose these times were still deterministic, but were instead say \( r \) days and \( s \) days respectively. All that would be necessary is to adjust certain upper limits of summation in the supply and demand constraints. For example, \((k-1)\) would be replaced by \((k-r)\) in the sum over \(x_{\text{min}}\) in Eq. (2).

A more complicated case is that of a probability distribution of times to traverse the arc (n, i) or a distribution of times to unload a car. One way to handle this would be to include, on the right-hand side of (2), a “safety stock” of cars at i in addition to the cars already there. A better way is available, however. It is more in keeping with the DRP philosophy to add a “safety lead time”[23] of, say, one day. Dispatch the cars from node n one day earlier than before. If all went well, they would arrive at i one day sooner than actually needed, but if not, there is an additional day to deal with contingencies. In any event, there are no extra cars at i.

We notched in the introduction the increased emphasis on inter-modal services, i.e. trailer-on-flatcar. The two major Canadian railways offer such services themselves, and our model can be applied to manage the inventories of trailers. As before, \( x \) and \( y \) would denote empty and full units, etc.

The approach of this paper has been entirely from the point of view of a railway. In fact, DRP and the transshipment model can also be used by a producer of bulk commodities to manage its private fleet of rail cars. We have recently begun such an application; it will be the subject of a subsequent paper.

REFERENCES

Application of Iterative Scaling Method to Calibrate a Combined Trip Distribution and Mode Choice Model by Hsiao-Fan Wang

ABSTRACT
The Generalised Iterative Scaling Method of Darroch and Ratcliff (1972) is a broad-spectrum approach to calibrating trip distribution models (Wang, 1983); it is most significant when the function form of an deterrent effect is unknown. Existing applications show that if additional decision factors are considered sequentially, more dimensions of the model are correspondingly required. However, it has been recognised that the travel decisions of why, where, how and which route to follow are made simultaneously. Therefore, this study first extends the current 3-dimensional procedure (Evans and Kirby, 1974) for trip distribution problem to a combined trip distribution and mode choice model, and second, shows that only if the 3-dimensional approach is employed can the model be representative of simultaneous decisions. The calibration and prediction procedures are specified and a numerical evaluation of the model is provided.

I. Introduction
Darroch and Ratcliff's (1972) Generalised Iterative Scaling Method was developed for log-linear models with both cyclic and non-cyclic approaches; it has been proven that, if marginal constraints are satisfied, a unique n-dimensional contingency table can be obtained. Wang (1983) then applied this to estimate the values of trip distribution matrices through three existing models, namely, the Furness model, the doubly constrained gravity model (which uses a cyclic approach), and the Detroit model (which uses a non-cyclic approach). Wang thus showed that the conventional 2-dimensional Furness procedure, 3-dimensional Furness procedure for gravity models (Evans and Kirby, 1974), and Detroit model are special cases of the Iterative Scaling Method.

In terms of transportation planning, the basic difference among these three models, when trips are distributed between traffic zones, is that the 2-dimensional Furness procedure and Detroit method are classified as Growth-