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MANUSCRIPT for the Journal of Operations Management

Production Planning for Mixed Assembly/Arborescent Systems

James H. Bookbinder
Leslie A. Koch

EXECUTIVE SUMMARY
In the automotive industry, components are fabricated from raw materials; modules from components; and subassemblies from modules. Final assembly of the end item, the vehicle, then follows. Such a process would be called an "assembly system" if each lower-level entity (raw material, component, module) were used in only one immediate successor entity (component, module, subassembly).

It is common to represent the bill of materials as a directed network. The nodes of this network are entities in the bill of materials; arcs in the network point from lower level entities in the direction of higher level entities and toward the end product. For example, if components 1 and 2 were combined to produce module a, there would be an arc from node 1 to node a. Similarly, there would be an arc pointing from node 2 to node a.

A pure assembly system may thus be described by saying that each node has at most one direct successor, although it may have several immediate predecessors. Similarly, a pure "arborescent network" is the prototype for a distribution system, whereby each node (say a warehouse) may have at most one immediate predecessor (distribution center), but possibly many immediate successors (retailers or smaller warehouses).

The focus of this paper is that a complex manufacturing system often has arborescent or non-assembly portions in its network. In automotive assembly, a given component frequently appears in more than one subassembly in the manufactured product. Certain components are used in each of four wheels; left- and right-side bore subassemblies contain common modules. The result is a mixed assembly/arborescent structure. Mixed assembly/arborescent structures also merit consideration as the prototype for combined production/distribution systems.

In this article, we consider such mixed structures by building on properties of pure assembly and pure arborescent systems. First, for a wide class of parameters and various pure assembly structures, we study several single-level lot sizing algorithms in conjunction with the cost parameter-revision method of Blackburn and Milten (1982b). Cost-parameter revision is a way to account for the impact at other stages, of decisions made at a given stage, in a pure assembly system. We also show it is better not to revise these cost parameters for a pure arborescent system.

Our approach to a mixed assembly/arborescent system is thus based on identifying its "largest, independent pure assembly sub-graph." This is a significant subsystem of the original graph or network, the largest portion that is pure assembly in nature. Modification of the cost parameters there, followed by stage-by-stage applications through the whole network of the best one-stage lot sizing method, gives lower total costs than when no cost revision is applied in the mixed system. Suggestions are then made for further research.

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INTRODUCTION

It has long been appreciated, by both practitioners and academics, that decisions made at one level in a multi-stage production system can strongly impact results at other levels. Consider the fabrication of a component A at stage 2, for assembly (with other components) at stage 1. If stage 1 releases orders too frequently, or often revises orders already released, nervousness will result at stage 2. On the other hand, if stage 2 cannot reliably get a short enough lead time from its supplier of raw materials, stage 1 may occasionally have to shut down or delay the assembly process.

The preceding examples pertain to the impact at one stage of timing on another stage. However, if stage 1 mis-specifies the quantity of components (or stage 2 the quantity of raw materials), the result could be the same. Even if there is not a stockout or shutdown, the total cost performance of the system can be degraded without sufficient coordination between the two stages.

The present article is concerned with such coordination in certain resource-unconstrained, multi-stage systems ("assembly/arborecent," as explained below). The mechanism suggested for this coordination (Blackburn and Millen (1982b)) is to permit decentralized decision making. That is, once a certain procedure called "cost-parameter revision" is carried out in the appropriate portion of the overall network, these revised costs can be used to make somewhat independent lot-sizing decisions at each stage of the system. Cost-parameter revision is thus a way to take into account the impact at other stages, of decisions made at a given stage. Reasonable solutions will be seen to result.

Systems of the mixed assembly/arborecent type arise in practice when some components, modules, etc. in the bill of materials are used in more than one higher-level item. In addition, a mixed assembly/arborecent system is representative of a network for combined production and distribution operations.

Organization of this Paper

This paper is organized as follows. After a literature review, we begin by studying several pure assembly structures. Four single-level lot-sizing heuristics will be applied stage-by-stage to a number of static twelve-period problems. Besides allowing an illustration of the cost-parameter revision method (the details of which are in the Appendix), this will also furnish some basis for selection of one of these heuristics for use in the following sections. There we consider arborecent systems and a variety of mixed assembly/arborecent structures of varying degrees of complexity. It will be seen that the key to good cost performance for a mixed structure is the identification of the "largest, independent pure assembly sub-graph" in the original structure. We then conclude this paper with a summary of our findings and suggestions for further research.

LITERATURE REVIEW

There is a well known and extensive literature for the problem of production planning in the case of time-varying demands. When these demands \(d_1, d_2, \ldots, d_T\) are deterministic, and there is a single stage of production and a static time-horizon \(T\), the Wagner-Whitin (1958) method gives the optimal solution, whatever may be the demands \(d_j\), as long as they are known in advance. Optimality results because the WW algorithm is based on an exact dynamic programming solution. While it has been hard to implement this methodology in industry, the outcome might have been different had personal computers and user-friendly software already existed.

Implementation has generally been more straightforward to achieve with a heuristic method. A review by Richie (1986) considers mostly the cases in which there is either an increasing or decreasing trend in demand, for a single stage of production. One of the best known heuristics is that of Silver-Meal (SM) (1973). Bookbinder and Tan (1983) developed two heuristics (H1 and H2) which often performed as well as SM, sometimes better, when tested against a number of heuristics in either a static horizon (Bookbinder and Tan (1983)) or a rolling schedule (Bookbinder and H'ng (1986)). Zoller and Rebraca (1988) have also recently conducted extensive tests of lot-sizing heuristics commonly embedded in commercially-available MRP systems (Vollmann, et al. (1988)).

Bahl, Ritzman and Gupta (1987) have categorized, across five dimensions, a great many heuristics and optimum-seeking algorithms for determining lot sizes and resource requirements. Separate discussions are given for single-stage and multi-stage lot-sizing problems, each with and without resource constraints.

This paper is concerned with multi-stage production planning. For the multi-stage problem in a static horizon, the formulations of Love (1972) and Crowston and Wagner (1973) are optimal for an assembly system (Figure 1). One reason to consider the heuristics of the present paper is the question of whether near-optimal solutions could be obtained with less computational effort (Koch (1986)). A second is that we wish to study combined production/distribution systems. Note that, while an assembly system is the prototype bill of materials for the simplest pure manufacturing operation (say as in MRP), a distribution system is arborecent (Figure 2): each node may have only a single predecessor but perhaps multiple successors. The assumptions and methods of Love (1972) and of Crowston and Wagner (1973) do not apply to arborecent systems.

**FIGURE 1**
TWO SIMPLE EXAMPLES OF AN ASSEMBLY STRUCTURE, OR PURE MANUFACTURING SYSTEM

Each node can have only one successor, although there may be multiple predecessors (See also Figure 4).

---

1
2
3
Most heuristics for multi-stage planning, however, have been tested solely on assembly systems. In this regard, we cite the studies of multi-stage production by Choi, et al. (1984); Jacobs and Khumawala (1982); Lambrecht, et al. (1983); Wemmerlöv (1981) and Yelle (1978). These papers generally concerned the choice of which single-stage lot-sizing heuristic should be employed stage-by-stage at every echelon in the multi-stage structure, or whether a different heuristic should be selected for each echelon (Yelle, 1978). Wemmerlöv (1981) has discussed the pros and cons of using "echelon" holding costs in applying these single-stage heuristics.

Echelon costs, the net difference in the costs to hold one unit at each pair of consecutive stages in the multi-level structure, may be thought of as a "transfer price" (Blackburn and Millen, 1982b) to move a single unit from a given stage in the multi-echelon system to its immediate successor. Blackburn and Millen (1982b) thus take into account the impact elsewhere in the multi-stage structure of lot-sizing decisions at each given level. Their heuristic modifies the cost parameters at individual stages in order to reflect the demand dependency between stages. This cost modification was found by Blackburn and Millen to work well for both a fixed time horizon (1982b) and for a rolling schedule (1982a). However, their assumptions limit this approach to the case of pure assembly systems.

Bookbinder and Koch (1980) have developed a new formulation for multi-stage production lot sizing (see also Koch, 1986). That model would furnish an exact solution, when there are deterministic time-varying demands, in a multi-stage system of whatever structure. However, the methodology requires solution of a non-linear 0-1 programming problem, and it is not yet clear whether the computations involved will permit routine implementation in practical situations. It is thus appropriate to seek a simpler heuristic procedure more likely to be adopted in industry.

While the mixed assembly/arborecent structure is not common in the literature on manufacturing, it should be: a mixed assembly/arborecent structure recognizes that a given component may appear in more than one subassembly in the manufactured product. As well, the mixed structure merits consideration as the prototype for integrated production/distribution systems. The approach which we propose in this article would be particularly suited for the mixed structure, whose distribution operations thus comprise the arborecent portions of the network. (See Cohen and Lee (1988) for a discussion of the strategic aspects of integrated production/distribution systems.)

It should be kept in mind in all that follows that we treat solely the case of a static or fixed time-horizon T, not that of a rolling schedule (Blackburn and Millen (1980, 1982a), Bookbinder and Heath (1988), Bookbinder and H'ng (1986)). Moreover, although we deal with time-varying demands and there are no restrictions on the degree to which demand can change from one period to the next, these demands \( d_n \) are entirely deterministic. The \( d_n \) are known at time zero for every period \( n \leq T \) in the planning horizon. There are thus no issues here regarding the probability distribution of demands nor of a service-level constraint (Bookbinder and Tan, 1986).

**ASSEMBLY STRUCTURES**

**Introduction**

As mentioned earlier, Blackburn and Millen (1982b) modify the setup cost and the echelon-cost parameter at each stage in a multi-echelon system. This reflects the influence at other stages of lot-sizing decisions at a given stage, yet permits an individual decision to be analyzed with a
single-stage lot-sizing algorithm. To place in context our experiments with arborescent and mixed assembly/arborescent systems, we study in this section several pure assembly structures (Figure 4). The cost-modification procedures were developed for assembly systems, and this is where Blackburn and Millen's (1982b) assumptions apply. We will thus get a feel for the influence on their method of the choice of single-stage lot-sizing heuristic, and perhaps see how the depth (Figure 4a) versus breadth (Figure 4b) of the multi-stage structure affects the results.

FIGURE 4
PURE ASSEMBLY STRUCTURES TESTED

(a)

(b)

(c)

Once the cost parameters are modified, stage-by-stage application of the following four single-stage lot-sizing algorithms will be carried out in turn: H1 and H2 of Bookbinder and Tan (1985); the Silver-Meal (1973) heuristic; and the Wagner-Whitin (1958) algorithm. H1 and H2 were originally introduced by Bookbinder and Tan to deal with cases that SM identified as difficult: a decreasing trend in demand, and periods of zero demand. H1 is based on modifying the stopping rule of SM. H2 attempts to combine the merits of SM with those of the least-unit-cost heuristic.

Many other single-stage heuristics could have been selected for study in the present paper. A rationale for the four above, and other possible choices, are given in Bookbinder and Heath (1988), Bookbinder and H'ng (1986) and Zeller and Robrade (1988). We also remark that, in this context, the WW algorithm should be considered a heuristic, rather than necessarily exact. WW furnishes the optimal solution for a single-stage static problem, but optimality need not result once the several stages are "decoupled."

Experiments
Our first experiments are thus as follows. We test independently the three assembly systems of Figure 4. For each lot-sizing problem, the setup cost for a given stage is randomly selected from the set {50, 75, 80, 100, 175, 300, 400, 450, 500} and echelon holding costs at a particular stage randomly chosen from among {0.1, 0.5, 1, 2, 4, 8}. The resulting variation in the ratios of setup to holding costs thus provides a wide range of "natural cycles" or replenishment intervals. While other choices of setup cost could have been included in the same range 50 to 900, the resulting thousands of distinct problem instances constitute a balanced set of cost-parameter combinations. This is because random selection of cost parameters was performed four times, independently, at each stage. (Note that there are five or ten stages in each system of Figure 4).

For each of the three assembly systems, one hundred static twelve-period problems were randomly generated for each cost-parameter set. The same problems were solved by each of the four lot-sizing heuristics. Any particular problem instance had end-item demand in each period drawn from the Uniform distribution on [0, 200]. Each set of those demands (4 times, once randomly drawn, was then assumed known at time zero for all t, 1 < t < 12).

In summary, the number of (assembly-structure) production plans to be developed using the cost revision technique was thus 4x100x4x5x12x4x5 = 40,000. (The terms in parentheses reflect the fact that Figures 4a and 4b have more nodes than does Figure 4c). That is, 10,000 distinct problem instances are each solved by four lot-sizing heuristics. After obtaining a production plan for each end-item demand and the other parameters which together constitute one problem instance, the total costs (setup plus inventory-carrying) can be compared across single-stage lot-sizing algorithms.

This comparison is most efficiently made (Bookbinder and Heath (1988), Bookbinder and H'ng (1986) by studying cost differences, i.e., the total costs of each lot-sizing heuristic minus the costs of the Wagner-Whitin algorithm. These mean differences are calculated over the 100 different demand patterns and averaged over the four sets of cost parameters. Results are given in Table 1 in terms of a 90% confidence interval for the cost differences.

There are nine confidence intervals, representing the three independent choices of lot-sizing heuristic and three assembly structures. Only one confidence interval contains zero: there was no significant difference in the cost performances of H2 and WW for assembly structure a. All other cost differences were statistically significant at the 10% level. There is thus some basis for saying that WW was the best heuristic to use with the cost-revision technique, at least for the particular assembly structures a, b, c of Figure 4. Naturally, there may be some dependence in the rankings here on the "realized" values of the setup and holding costs. It is interesting, however, that the relative superiority of WW and H2 with respect to H1 and SM is consistent with the greater mathematical effort of the former two relative to the latter.

ARBORESCENT SYSTEMS

As mentioned above, and as Blackburn and Millen (1982b) recognized, the assumptions behind the cost-revision method do not hold for an arborescent system. In this section, we show that cost-parameter revision need not yield improved cost results for the arborescent structure. Rather, the following example shows that cost revision may be inferior to "no revision" in the case of arborescence.
TABLE 1
MEAN COST DIFFERENCES, AND 90% CONFIDENCE INTERVALS,
FOR THE PURE ASSEMBLY STRUCTURES a,b,c GIVEN IN FIGURE 4

<table>
<thead>
<tr>
<th>Cost Difference</th>
<th>Mean Cost Difference</th>
<th>Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>a: SM - WW</td>
<td>695.6</td>
<td>(600.7, 786.5)</td>
</tr>
<tr>
<td>H1 - WW</td>
<td>657.8</td>
<td>(261.1, 1029.5)</td>
</tr>
<tr>
<td>H2 - WW</td>
<td>21.5</td>
<td>(-18.9, 62.1)</td>
</tr>
<tr>
<td>b: SM - WW</td>
<td>349.9</td>
<td>(280.2, 419.6)</td>
</tr>
<tr>
<td>H1 - WW</td>
<td>736.9</td>
<td>(671.3, 795.5)</td>
</tr>
<tr>
<td>H2 - WW</td>
<td>370.8</td>
<td>(313.6, 428.0)</td>
</tr>
<tr>
<td>c: SM - WW</td>
<td>1279.5</td>
<td>(1203.9, 1337.1)</td>
</tr>
<tr>
<td>H1 - WW</td>
<td>350.2</td>
<td>(491.7, 608.7)</td>
</tr>
<tr>
<td>H2 - WW</td>
<td>135.0</td>
<td>(32.8, 187.2)</td>
</tr>
</tbody>
</table>

Costs are in units of £1000, the confidence intervals are at a 90% level.

We thus find the revised set-up costs to be $AR_1 = AR_3 = 27$, while the revised echelon holding costs are $e_2 = e_3 = 0.3$.

Since $k_4$ and $k_5$ are integers, the optimal solution would be expected when applying the Wagner-Whitin algorithm to each individual stage (Blackburn and Millen 1962a,b). Table 2 presents the production schedule resulting from the use of cost revision. (Having applied cost revision to obtain this production schedule, its total costs are most easily calculated based on the original parameters $A_i$ and $h_i$). Table 3 shows a superior production plan and total costs.

TABLE 2
ARBORESCENCE WITH COST REVISION

<table>
<thead>
<tr>
<th>Stage</th>
<th>Period</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>End Inv</td>
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<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
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<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
</tbody>
</table>

Cost revision did not produce better results because equation (3) is not well defined for an arborescent structure: note 4 (Figure 2) has more than one successor. Separate consideration of the stages (2, 4) and (3, 4) did permit application of the equation (3) for the $k_4$. However, this amounts to treating node 2 and node 3 independently, whereas their combined impact on node 4 is more important but neglected. Most of the improvements in Table 3 results from synchronizing the reorder intervals of nodes 2 and 3. This allows grouping their requirements on node 4 as two orders of equal magnitude.

We thus find the revised set-up costs to be $AR_1 = AR_3 = 27$, while the revised echelon holding costs are $e_2 = e_3 = 0.3$.

Since $k_4$ and $k_5$ are integers, the optimal solution would be expected when applying the Wagner-Whitin algorithm to each individual stage (Blackburn and Millen 1962a,b). Table 2 presents the production schedule resulting from the use of cost revision. (Having applied cost revision to obtain this production schedule, its total costs are most easily calculated based on the original parameters $A_i$ and $h_i$). Table 3 shows a superior production plan and total costs.
TABLE 3
SUPERIOR PRODUCTION PLANS AND TOTAL COSTS FOR THE ARBORESCENT SYSTEM OF FIGURE 2

<table>
<thead>
<tr>
<th>Stage 2</th>
<th>Period</th>
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<th>2</th>
<th>3</th>
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</tr>
</tbody>
</table>

Total Cost = \( \sum_{i=2}^{4} (\text{Inv Carry Costs, plus Setup Costs, at Stage } i) \)

= 60 + 84 + 126

= 270

MIXED ASSEMBLY/ARBORESCENT STRUCTURES

Introduction

The fact that cost revision works well for assembly structures, but not for arborescent structures, motivates our proposed heuristic for the mixed assembly/arborescent system. We suggest revision of the cost parameters only for the assembly subsystem in a mixed assembly/arborescent structure. If there exists more than one (independent) assembly subsystem, this cost revision should be done separately for each such substructure. One then applies, stage-by-stage, with cost parameters modified as appropriate, WW or perhaps some other single-stage lot-sizing heuristic.

It should be recalled that no optimal-solution method has been published for a multi-echelon system with time-varying demands. In that light, it is not unreasonable that we suggest a "decentralized" approach: independent application at each echelon of a good single-stage method (WW). Suitable modified costs permit independent application in our method. For comparison, WW will be independently applied with no cost revision whatever.

The Experiments

We will experiment with three mixed manufacturing structures, whose complexity increases from structure one to structure three (Figures 3, 6, and 7). The first structure, the simplest possible assembly/arborescent system, provides the framework for our proposed heuristic. The more complex structures serve to test the effectiveness of the newly devised heuristic. These particular structures were chosen with reference to certain subassemblies in the automotive industry. Many other choices of mixed networks are possible.

For each structure, one set of cost parameters is used, as well as the same 100 demand schedules in the experiments of the previous section. We compare our results to the stage-by-stage application of the Wagner-Whitin algorithm with no cost revision. WW was chosen since it produced the best results for the pure assembly systems (Table 1). As before, a paired t-test is used to analyze the effectiveness of our heuristic. Note that structure one requires solution of 1x2x4x100 = 800 problems, structure two 1x2x6x100 = 1200 problems and structure three 1x2x1x100 = 2200 problems.

Figure 3 represents the most basic mixed assembly/arborescent manufacturing system. As mentioned, we propose that cost-parameter revision be applied only to the pure assembly substructure. In this case, that assembly subsystem consists of stages (1, 2, 3). Node 4 cannot be included; by definition, a node in an assembly structure has at most a single immediate successor, although it can have more than one predecessor. In Table 4, we exhibit the cost differences for the mixed structure 1 (Figure 3). The results are statistically significant at the level \( \alpha = 0.10 \), and indicate that our heuristic yields better cost performance than the stage-by-stage application of WW with no cost-parameter revision whatever.

TABLE 4
COST RESULTS FOR THE MIXED ASSEMBLY/ARBORESCENT STRUCTURE 1 (FIGURE 3)

<table>
<thead>
<tr>
<th>Mean Cost Difference</th>
<th>68.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>90% Confidence Interval</td>
<td>(61.6, 76.0)</td>
</tr>
</tbody>
</table>

More Complex Mixed Structures

To illustrate the considerations necessary when there is more than one assembly substructure, consider the mixed assembly/arborescent system of Figure 5. In our procedure, one simply removes from the system graph any node(s) with more than a single immediate successor. Beginning at the bottom of the graph, at the highest numbered stage, the structure is cut horizontally just above each such node with two or more immediate successors. The lower remaining portion may be an independent assembly sub-graph (Figures 7, 6, 5) in Figure 5) or it may not be (Node 7 alone in Figure 6, or nodes 7, 5 in Figure 6). We have found that the concept of an "independent assembly sub-graph" makes sense only if at least one node in the sub-graph has less than two successors in the original structure. For example, if in Figure 6, there were a node 8 whose only connection to the network were as an immediate predecessor of node 7, then we would treat (8, 7) as an independent assembly sub-graph with two nodes.

Figures 5, 6 and 7 show some mixed structures and the resulting independent assembly sub-graphs. In Figure 5, the horizontal cut just above node 5 yields two distinct assembly sub-graphs whose costs are thus revised independently. We also remark that, for cost-revision purposes, a given node cannot be part of more than one assembly sub-graph.
In summary, the second assembly/arborescent structure for our experiments (Figure 6) involves two arborescent stages (nodes 5 and 7) which will be ignored as the costs are revised for the remaining pure assembly subsystem (nodes 6,4,3,2). Our third mixed structure (Figure 7) contains an arborescent stage (node 10) which is the demarcation point between the two independent sub-graphs. In Figures 6 and 7, and in any application of our procedures, once the revisions have been made to the appropriate setup and echelon costs, the lot-sizing calculation is then done stage-by-stage in the original, complete structure with the newly-revised costs.

Further Results for Mixed Structures

Table 5 presents our results for the mixed assembly/arborescent systems of Figures 6 and 7. Note that no confidence interval contains zero. These results, plus those of Table 4, show that cost savings are statistically significant at the 10% level, when the cost-revision method (Blackburn and Millen [1982b]) is applied to each independent, pure assembly sub-graph in the mixed-structure environment. These cost savings are typically 5% of total costs (inventory carrying plus setups).
CONCLUSIONS AND FURTHER RESEARCH

In this paper, we have studied a number of assembly/arborescent systems by building on the properties of the related structures, pure assembly and pure arborescent systems. First, we showed (Table 1) that for a wide class of parameters and various assembly structures (Figure 4), the Wagner–Whitin algorithm works best, when applied stage-by-stage in conjunction with the cost-parameter-revision method of Blackburn and Millen (1982b). That method (see the Appendix) was of course developed for assembly systems, and not necessarily intended for other cases. Indeed, comparison of Tables 2 and 3 demonstrates that, for the pure arborescent system of Figure 2, lower total costs can be obtained if cost revision is not applied.

This led to our approach for mixed assembly/arborescent systems. For any mixed structure, one identifies the “largest, independent pure assembly sub-graph” (see Figures 5–7). Modification of the setup and echelon cost parameters here, followed by stage-by-stage application throughout the whole network of the best single-level lot-sizing method (WW), gave results superior to the case of no cost-revision (see Tables 4 and 5) for the mixed assembly/arborescent systems of Figures 3, 6 and 7.

Several possibilities remain for further research. The approach of this paper could be extended to mixed assembly/arborescent structures in a rolling schedule, and eventually related to the work of Blackburn and Millen (1982a), Bookbinder and Heath (1988) and Bookbinder and H’ng (1986). The case of probabilistic demands (Bookbinder and Tan, 1988) might also be interesting to study for mixed assembly/arborescent systems. This would test, for example, whether cost-parameter revision becomes more important or less so, when combined with concern for the service level achieved at each echelon.

Finally, it would be worthwhile to conduct a case study for a client organization. Bookbinder and Heath (1988) analyzed lot sizing in the context of a DRP system for a distributor of grocery products. Implementation of the approach of the present paper could be attempted for a company with a combined MRP/DRP system. Alternatively, in a pure manufacturing setting, the production-planning issues discussed above could be field-tested for a firm whose products have varying degrees of component commonality. As a benchmark, a product with no commonality is simply an assembly system, for which the methods of Blackburn and Millen (1982b) can be applied. Products whose bills of material have small, medium and large degrees of commonality can be studied by the methods of the present article.
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REFERENCES


APPENDIX

COST-PARAMETER REVISION

In this Appendix, we summarize the results of Blackburn and Miller (1982b) on cost-parameter revision. At each stage l in a multi-stage assembly system, the setup cost $A_l$ is revised to $A_{l+s}$ and the echelon holding cost $e_l$ is revised to $e_{l+s}$. Their expressions are:

$$A_{l+s} = A_l + \sum e_l \cdot AR_{l+s} \cdot k_l \cdot j^{l+s}(i)$$

and

$$e_{l+s} = e_l + \sum e_{l+s} \cdot k_l \cdot j^{l+s}(i)$$

in which the sum is over all nodes $j$ that are immediate predecessors of node l.

The factor $k_l$ is important to ensure compatibility of the respective production quantities at successive stages. Its value is found to be

$$k_l = \left( \frac{A_{l+s} - e_{l+s} \cdot j^{l+s}(i)}{A_l - e_l \cdot j^{l+s}(i)} \right)^{1/2}$$

where $S(j)$ denotes the (single) immediate successor of node $j$. Note that in the context of an EDQ model, one may also interpret $k_l$ as a ratio of natural cycles defined via echelon holding costs.

The result for $k_l$ was derived by differential calculus, and can be shown to yield a minimum in the total cost function. However, care must be taken in employing this solution: a stockout may result at stage $l$ if $k_l$ is not an integer. In our applications of the cost-parameter revision technique (Tibet 3-5), we circumvent this difficulty by using an approach which Blackburn and Miller (1982b) term the Continuous-Constraint-K method (CC). The integrality constraint is ignored, but if the $k_l$ obtained from equation (3) is less than 1, then the value $k_l = 1$ is used instead.

Although several other cost-modification methods were suggested by Blackburn and Millman, they found that KCC generally performed best. We chose the Silver-Meal or Wagner-Whitin algorithms to use in conjunction with KCC in part because of Blackburn and Millman's results that these single-stage heuristics worked well with the KCC method.

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