A model to co-ordinate logistics with other functions gives summaries of resource usage, shortage levels, stock levels (by product) and system-wide flows of finished goods.

Budget Allocation and Profit for Logistics and its Interfaces

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It is almost a truism in distribution or logistics management that total cost is what counts, and that the least-cost distribution plan can be found by trading off the costs of carrying inventory with those of transport or warehousing. The actual situation is not that simple.

Most firms find it hard to co-ordinate each logistics function (say transport) with the others (e.g. inventory decisions). Each logistics activity is organised somewhat autonomously of the others; the performance of each is controlled and evaluated individually. A transport manager, therefore, will not eagerly accept increased costs, even if still greater savings would accrue from inventory reductions. Also, a company's procedures for management accounting rarely acknowledge benefits in one segment of the firm, at the expense of costs in another. Most single-objective OR models will simply subtract the latter from the former, but the accounting system will not record the difference in a meaningful way. The transport department will thus not be praised for helping reduce total logistics expenses, but rather criticised for not keeping down the costs of transport.

On the academic side, there are models to optimise particular aspects of logistics activities, but no model is available that would permit the Director of Logistics to allocate his/her overall budget between transport and inventory (thus removing centrally some of the preceding conflict). In fact, no model exists to aid the logistics Director in deciding an overall budget level to request from senior management. It is the purpose of the present article to develop such a model, to help answer these questions.

Systematic trade-offs between inventory and transport require an "integrated" model, representing two or more logistics functions and their interactions. Only a few such optimisation models appear in the literature (see[1] for a review). Examples include Blumenfeld et al.[2] and Arthur and Lawrence[3]. These and other published models (discussed in the following section) appear incomplete. They do not consider the budget level for logistics (except perhaps as a constraint). They omit the impact of customer service on future sales, and the possible loss of customers when demands cannot be delivered in a given period.

We propose a mathematical programming model to co-ordinate logistics decisions with those of their major interfaces (Figure 1): production planning and marketing. It will specify when to purchase raw materials, and how much; where, when and in what quantity to produce the products, and to which distribution centre(s) they should be shipped; and to which customers these goods should ultimately be sent.

Long-term facilities (plants, warehouses) are assumed fixed in size and location, hence our model contains no integer variables. This linear programme attempts to "design an improved logistics system", instead of "optimising a given logistics system". The "given system" is one whose raw materials and budget would be specified inputs. However[4, pp. 338-44], there can be striking benefits to treating resources as LP decision variables. We thus let the budget for expenditures on logistics and its interface activities be an objective to minimise. We also wish to maximise the profit of logistics and its related interfaces.

It should be kept in mind throughout that our profit objective includes more than just tangible revenues and
costs. “Profit” in our model contains several penalty or opportunity costs. (See the final section of the Appendix for their definition and the reasoning behind them.) Without these intangibles, the budgeted costs would automatically be considered in a maximisation of profit. It is the intangible costs which thus ensure a trade-off between budget and profit.

Those trade-offs between the two conflicting aims will be made using the GPSTEM method[5,6] to obtain the “best compromise” solution[4]. An important part of this solution is thus a suggested overall budget level for logistics. Such a request to senior management would have strong justification.

Among its outputs, our interactive model will determine each period’s inventory levels (for each product at every depot; for all raw materials at each plant) and system-wide transport flows. The Director of Logistics will thus obtain the optimal allocation between inventory and transport of the preceding budget.

Numerical results are presented later in the article. To set the stage we next review some relevant logistics literature. Attention is confined to single- or multiple-objective optimisation models whose cost parameters involve both sides of an interface in Figure 1.

Pertinent approaches to multiple-criteria decision making are discussed next, details of the model itself are then presented. We conclude with ideas for further research.

**Review of Logistics Models**

Most optimisation models in distribution pertain to one activity such as transport, inventory or warehousing[7]. A few models, however, do treat more than a single logistics function. Geoffrion and Graves’s facility-location study[8] presents one of the first optimisation models to consider the combined cost of production, warehousing and transport. Osleeb and Cromley[9] treat only one product group and no distribution centres, but they do minimise the sum of costs of transport to customers plus the (non-linear) production costs.


The preceding distribution models[2,3,8,10] each have strong points. However, most such optimisation models do not account for the importance of the budget level in logistics. Those which do include the budget[3,10] accept it as an input constraint (the budget will be an output of our model). In addition, all available models take demand as exogenously specified. These models do not directly include the effect of customer service on future sales, nor the possible loss of customers whose demands cannot be delivered in a given period. As well, some models do attempt to include production, but only by taking the quantity produced as equal to demand, or by specifying a strict level of resource usage.

In this article, we will deal with the key issues of the previous paragraph by developing an integrated logistics model, representing several functions and their interactions. We will thus emphasise the box outlined in Figure 1[7].

To lay the groundwork for our multiple-objective model, we first offer a brief survey of methods appropriate to decision making with several criteria.

**The GPSTEM Method for Multicriteria Decision Making**

In deciding how to solve our model, we considered the following interactive multiple-objective approaches: the STEM method[11], the method of the displaced ideal[4], the GPSTEM method[5] and the vector-maximum algorithm[12]. However, it turns out that the STEM and GPSTEM methods can attack problems of considerably greater size.

The GPSTEM method improves upon Goal Programming (GP) in that goals in GPSTEM are based on knowledge
of the ideal solutions, furnished by the initial phase of this method. Moreover, the decision maker (DM) can relax the goals systematically by using parametric programming.

As for the STEM method[11], one drawback is that in its first iteration, the compromise solution may be very far from the DM's satisfactory levels (goals). GPSTEM, however, permits exploration of non-dominated solutions whose individual objective values are close to those goal levels. In [13], the Goal Programming Step Method (GPSTEM)[5] tries to eliminate the weaknesses of the GP and the STEM methods.

Occasionally (in the real world of logistics), there can be complete disagreement among several managers when each represents one objective of the multiple-objective problem. Consequently, the GPSTEM method also provides the equilibrium solution $x^*$ to a particular non-zero sum, non-co-operative two-person bi-matrix game[4]. This equilibrium solution is a viable compromise in the event of such disagreement.

We will thus employ the GPSTEM method[6] to solve several numerical examples of our logistics model. The following remarks pertain to this solution process and the underlying model philosophy.

The logistics function and its interfaces both employ capital, incur cost and support sales through the customer service level. We thus treat logistics plus its interfaces as a "separate business unit" which contributes profit to the firm. Realised profit will enable the executive to measure performance of the managers of logistics and production, and will furnish an additional means of control beyond that of the budget.

The executive (DM) initially confronts two conflicting aims for logistics and its interfaces. One is to obtain as large a profit as possible for this business unit (the objective of the logistics manager), while the other is to minimise the associated budget expenditure (the objective of the financial manager). Step 1 of GPSTEM has the executive begin by finding the maximum profit and the minimum budget, solving the model individually for each objective. These are the ideal solutions. Their values enable him or her to specify goals (satisfactory levels) for each objective in Step 2, involving goal programming.

After that step, the DM will review the actual progress towards those goals. Suppose the deviations from the levels initially given were acceptable for the budget but not for profit. The finance manager will then be asked to consider relaxing the budget constraint, by an amount suggested by the executive (with the aid of parametric programming, Step 3 in GPSTEM), to allow an increase in profit.

Alternatively, deviation from goal may be acceptable for profit but not for budget. The executive may then similarly decide on a (parametric) relaxation in profit, hoping to improve the budget required. We remark that one output of the model is the "interval of relaxation" for each goal. For the iterations of GPSTEM to continue, at least one goal must be relaxed by an amount contained in its respective interval of relaxation.

This procedure will generate a series of compromise solutions until the DM's satisfaction with the level of both objectives. However, suppose s/he is unsatisfied by the results, yet the logistics and financial managers do not want to relax their goal levels. A three should then accept the equilibrium solution $x^*$ given by the model, where relaxation of those goals is imposed by the algorithm itself.

**Formulation of the Model**

**Basic Assumptions**

We thus propose an interactive model to plan the manufacturing and shipping patterns of a multiproduct firm, from its plants through several distribution centres to its customers. The assumptions of this model are as follows:

- the model is deterministic with no economies of scale;
- it is an intermediate-range planning model. Issues such as vehicle routing and work-in-process inventory (i.e., short-term decisions) or plant location (long-term decision) are not considered;
- purchasing, production and distribution activities, all determined by the model for each period, are entirely carried out within that period;
- back-orders from the previous period are added to the present period's demand;
- an increase in resource costs during a period will influence production and inventory costs at the beginning of the next period;
- the plants hold inventory only of raw materials, not finished goods.

A detailed analytic statement of the model's variables, objective functions and constraints is given in the Appendix. We next discuss the effect of lost sales.

**Effect of Lost Sales**

Most models treat the distribution problem as one of cost minimisation. This standard approach cannot take into account the effect of lost sales on future revenue. It prescribes demand throughout the planning horizon, although it should not. An important consideration for our model is thus a desirable customer service level, desirable in the sense of reduced lost sales.

We will classify demands as either "potential" or "effective". Prespecified, expected demand is termed "potential demand". A shortage in one period requires
that at the start of the next, each customer’s potential demands for that product be adjusted by subtracting from it the lost sale, and by adding to it any backorder of the previous period. The resulting demand is called “effective demand”. See equation (5) of the Appendix.

In summary, an out-of-stock situation has three different impacts in our model. There is first a reduction of revenue in the given period. Moreover, the system experiences an increased shortage cost. Finally, demand in the current period will influence future periods’ potential demands (as above).

High shortage levels will lead to increased lost sales and back-orders. Realised sales will be lower than anticipated because effective demand will be less than potential demand. With such knowledge, the executive can balance revenue against the costs incurred in providing service. S/he will accept decreased demand, or will raise the budget goal and try to obtain the original demand level.

Data Requirements
Examination of the detailed model formulation in the Appendix reveals the necessary data to be of three types. The high majority is readily available. Examples include the transport costs and the purchase cost per unit of each resource.

In the second category of data are parameters where experimentation will be needed to fine-tune the model. Here we have in mind parameters such as the upper and lower bounds on resource and finished goods inventories, and the penalty cost of shortages (see the last two paragraphs of the Appendix). Following this experimentation, the user will be better able to understand the model’s results and to move them in the direction of trade-offs desired.

Typical of the third data type is \( Z_{im} \), the proportion of unfulfilled demand of item \( i \) that will be lost as a sale to customer \( m \). \( Z_{im} \) are the key parameters in equation (5). Sample surveys and follow-up interviews would help estimate these parameters. Although there may be some difficulty in the calibration of \( Z_{im} \), empirical effort here would be well invested. Naturally, in the numerical examples which follow, specific values have been input for \( Z_{im} \) and all other data.

Sample Solutions of the Model
Introduction
We now consider several examples which indicate how our two-objective model can deal smoothly with distinct mixtures of costs pertinent to three different firms. In Table 1 relative costs of production, transport and inventory (storage + opportunity cost of capital + stockout cost) are given for each firm.

The examples all have one plant, three resources, two products, two warehouses, three customers and three periods. Each problem is first solved using a single linear objective (profit maximisation) with a fixed $300,000 budget given as input. Each is then reanalysed using the two-criterion model and the GPSTEM method.

Whether as a single-objective model, or in the various steps of the GPSTEM method, each problem involves 96 to 100 decision variables and 90 to 92 constraints. We remark that in the numerical results which follow, the abbreviation \( K \) denotes thousands of dollars. We have often rounded larger figures to the nearest number of integer \( K \).

Results for each firm will be discussed in turn. A narrative description of successive iterations of the model will include their possible interpretation in each organisation. We remark that although these three examples resemble each other in the first step of GPSTEM, the dialogue for each example shows a variety of management reaction to alternative numerical trade-offs. GPSTEM can thus lead to a compromise solution or to the efficient equilibrium solution. These results are summarised at the end of the section, in Table II, to which the reader may wish to refer now, for a quick preview of the narrative.

Analysis of the First Firm’s Results
Profit maximisation. When the first example is solved with a given $300,000 budget, the profit is 103K. The optimal allocation of that budget is $170,000 to the Production Department and $130,000 to Logistics. The latter allocation is divided between Transport (from plant to warehouses, 48.5K; from warehouses to customers, 75.5K) and Inventory (5.9K).

GPSTEM solution of the bi-criterion formulation. As a second stage, suppose the financial manager desires a lower budget. The DM then finds the most appropriate levels of budget and profit, using the GPSTEM method.

In the first step, the objectives are optimised separately, and the ideal solutions for maximisation of profit and minimisation of budget are 134K and 13.6K respectively. When profit is at its ideal level, however, the budget is 375K; for the ideal budget level, the “profit” is −229K.

The DM (top manager) then specifies $100,000 as a goal level for profit and $250,000 for budget, just by examining

<table>
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<tr>
<th>Table I. Relative Cost Components for Each Firm</th>
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<tbody>
<tr>
<td>Production</td>
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<td>Transport</td>
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<td>Inventory</td>
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Entries in each column relate the cost elements of the given firm. (The ordinal scale does not permit inter-firm comparisons between different columns.)

* For this firm only, the resource costs/unit are higher in periods 2 and 3 than in period 1.
The preceding results. Going through the algorithm, the optimal parametric solutions $f(x^2)$ of step 3 are 71.7K and 250K for profit and budget respectively, and the efficient equilibrium solution of the bi-matrix game in step 4 is $f(x^*) = (103K, 300K)$. These results, $x^1$ and $x^*$, are thus proposed to the DM.

Suppose the DM finds $f(x^1)$ unsatisfactory, and accepts an increase of $300,000 in budget after analysing the possible interval of relaxation specified by the model. The new compromise solution $f(x^2)$ is 90.4K for profit, 280K for budget. The corresponding equilibrium points, and consequently the efficient equilibrium solution, do not change.

After reviewing $f(x^2)$ and $f(x^*)$ given above, the DM agrees to take $x^2$ as the final solution. The optimal distribution of the $280,000 budget is: Production department, 158K; Logistics department: (1) Transport: from plant to warehouses, 45.4K; from warehouses to customers, 70.3K; (2) Inventory costs: 5.9K.

The difference in profit (103K vs 90.4K) between the single-objective result and $x^2$ is of course due to the higher budget (300K vs 280K). Naturally, the figures in this example are for purposes of illustration. In an actual application, there is likely to be a much greater difference between the two levels of budget. An inappropriate budget will lead to an unachievable profit. Once a suitable budget is determined, however, it establishes the costs permissible for logistics and its interface activities.

### Results for the Second Firm

**The single-objective problem.** This firm has a maximum profit of 60.5K when the budget is unspecified as $300,000. While the total cost is the same as for Firm No. 1, the production costs are 15K lower and the transport costs, 15K higher. It is this increased transport expense that causes Firm No. 2 to have a lower profit, once the chain reaction leading to higher shortage costs is traced through the model.

The optimal distribution of the $300,000 budget is thus 155K to Production, and to the Logistics department: (1) Transport: from plant to warehouses, 44.0K; from warehouses to customers, 95.2K; and (2) Inventory: 5.9K.

**The multiple-objective case.** The executive feels the profit of 60.5K is unacceptable, and uses our bi-criteria model to find satisfactory levels of both profit and budget. Optimisation of each objective individually (step 1) results in an ideal solution of 976K for maximisation of profit (the corresponding budget is 412K), but a minimum budget level of 14.2K (corresponding to a profit of -229K).

The DM begins with goals of $85,000 for profit, $280,000 for budget. After the execution of goal programming (step 2) and the parametric LP (step 3), the initial compromise solution is found to be $f(x^2) = (49.6K, 280K)$ for profit and budget respectively. The efficient equilibrium solution (step 4), on the other hand, is 73.0K and 329K for the same objectives.

The DM may be unsatisfied by the profit achieved in that compromise solution. However, the interval of relaxation of budget (calculated by the model) is [62.3K, 85.8K]. Continuation of GPSTEM would thus require at least 342K, a 22 per cent larger budget. If this were unacceptable to the financial manager, the executive would adopt a smaller increase in budget, and accept the initial equilibrium solution: (73.0K, 329K).

In that case, optimal distribution of the $329,000 budget is 170K to the Production department, and to Logistics: (1) Transport: from plant to warehouses, 50.1K; warehouses to customers, 103K; and (2) Inventory costs: 5.9K.

As can be seen, the new budget permits a greater allocation to each department than in the $300,000 case. This decreases the shortage cost and increases the effective demands, leading to higher profit.

### Firm No. 3

**Maximisation of profit.** Finally, let us examine the third firm. Its maximum attainable profit is only 22.6K when the budget is fixed at $300,000. This is due to higher resource costs experienced in the second and third periods. These costs are respectively 200 and 300, if resource costs/units are indexed as 100 in period 1). Initially, that this period expense increases the production and inventory cost at the beginning of the third period. The model deals with this by allowing purchase of resources and production in the first period, up to a level given by the capacity constraints and the inventory upper bounds. However, since the budget is fixed, the model is unable to satisfy the entire demand, and this firm suffers the greatest shortages of the three examples we consider.

The $300,000 budget is allocated as follows: Production department, 192K; Logistics department: (1) Transport: from plant to warehouses, 38.6K; from warehouses to customers, 59.9K; and (2) Inventory, 9.1K.
Results of the GPSTEM method. Let us now suppose the executive employs our bi-criterion model to find the best compromise solution. The maximum possible profit is 68.9K, but this requires a budget of $523,000. The ideal solution of the individual budget minisation problem, however, is 15.6K, corresponding to a profit of -233K.

The executive specifies goal levels of 47.5K for profit and $300,000 for budget. The optimal solution of the parametric LP (step 3) is, as expected, the same obtained by the single-objective formulation of this very problem: \( f(x^1) = (22.6K, 300K) \). As well, the equilibrium solution for the bi-matrix game is \( f(x^2) = (37.3K, 329K) \).

The executive, in rejecting the optimal solution \( f(x^1) \), requests the possible interval of relaxation. That turns out to be [49.4K, 67.2K]. He chooses $49,400 as the amount by which to increase the budget, which is also approved by the financial manager. The parametric LP (step 3) is solved once more to yield the second compromise solution, \( f(x^2) = (43.6K, 349K) \). The efficient equilibrium solution is \( f(x^2) = (55.5K, 404K) \). After reviewing these results, the DM decides to accept \( f(x^2) \) as the final solution.

In the optimal division of the ultimate $349,000 budget, Production receives 228K. For the Logistics department, there is Transport (from plant to warehouses, 42.9K; from warehouses to customers, 68.9K) and Inventory Carrying (9.3K).

Conclusions
Table II outlines the three problems studied. Intermediate results, and of course the final solutions, will be communicated to the production and logistics departments. The production manager will have a summary of supply and related resource usage as well as shortage levels. In the logistics department, the inventory and transport managers will receive reports on stock levels (by product) at each distribution centre, and the system-wide flows of finished goods, respectively. Each manager can then plan on how to allocate his/her share of the total budget to individual activities. The executive can thus evaluate performance by comparing the total profit achieved and the budget used, to the levels originally specified.

Further Research
Several possibilities for future research are now presented. These include decomposition of large-scale problems; simulation of probabilistic demand; consideration of cash flows; lost sales as a third objective; and inventories at retailers.

The model runs discussed in this article had around 100 variables and 90 constraints. Each problem pertained to three customers and two products. An industrial application, however, may involve a hundred customers and two dozen products. Brute-force solution may fall short in linear programmes of this size (1,800 variables, 1,200 constraints).

Fortunately its ‘block-angular’ form permits our model to be decomposed[15] with respect to the number of products \( I \), yielding \( I \) smaller-sized problems which are almost independent. This decomposition becomes even more efficient as the number of customers grows.

The deterministic model of this article eventually asks the DM to choose between a compromise solution and the efficient equilibrium solution. Each could be studied by simulation to show their detailed consequences in a stochastic environment.

Output results from a probabilistic simulation model would indicate potential improvements in the systems under consideration, perhaps by suggesting new or modified constraints for the optimisation model or by testing the effect of safety stocks for some resources or finished products. Simulation would in any case permit a more informed choice between good, feasible solutions to the multi-objective problem. (We remark that Bookbinder[16] has also considered the combination of simulation with an optimisation model, in the case of strategic distribution system design.)

In specifying a budget for logistics and its interfaces, our model can analyse future changes such as increased prices of raw materials. However, no account is taken of the timing of payments and receipts. A better allocation of funds to production and distribution would consider the firm’s cash situation. If cash balance equations for each period were included in the model, the impact on profit and budget of the paying (earning) of interest on negative (positive) cash balances could be studied.

A budget insufficient to satisfy demand can lead to a stockout, even in our deterministic model. Its treatment of lost sales, by subtracting a penalty cost from profit, is reasonable but can be improved. We suggest the use of “lost sales” as a third (minimisation) objective, whose best compromise level would be a trade-off with the budget. For example, a decrease in lost sales will likely increase the profit, but also require a greater budget.

We have assumed that, since customers (retailers) are not branches of the company under consideration, no account need be taken of retailer’s inventory positions. The model simply indicates how demands there should be satisfied (stocks replenished).

However, if retail outlets are controlled or even influenced by the firm, it may be reasonable that the model specify the customers’ stock levels. In that case we will have a multi-echelon inventory system: resources at the plants, finished goods at distribution centres and a final echelon of finished goods at customers.

Distribution Requirements Planning (DRP)[17,18] deals with inventories at multiple locations, and the timing and
quantities of goods shipped between echelons. The model of this article could be an input to the detailed transport plan of DRP. This would take account of lead times between factories and warehouses, and between warehouses and retailers, on a timescale shorter than one period in our model. (The latter is anticipated to be a month, whereas DRP would plan on a weekly basis.)

We remark that the idea of relating our model to DRP is analogous to obtaining, from an aggregate production plan, the corresponding detailed schedule. In this light, our model may be thought of as an aggregate plan for purchasing/production/distribution.

References

Appendix: Detailed Mathematical Formulation

Model Structure
Production of the \( I \) items occurs at \( J \) plants. Products are then shipped to \( K \) warehouses to meet the demand of \( M \) customers over an \( L \)-period planning horizon. Trade-offs are made between maximum profit and necessary minimum budget in the multi-objective optimisation model.

Unless stated otherwise, all sums over \( i, j, k, l, m, r \) cover the respective ranges \( 1 \leq i \leq I, 1 \leq j \leq J, 1 \leq k \leq K, 1 \leq l \leq L, 1 \leq m \leq M \) and \( 1 \leq r \leq R \), where \( R \) is the number of the resources input to the production process. Similarly, the "free" variables \( i, j, k, l, m, r \) when not summed, may take any value in the respective ranges.

Decision Variables
The following decision variables are defined, in which the "mnemonic" symbols \( PD \) and \( DC \) refer to shipments between plant and distribution centre, and from distribution centre to customer.

\[
P_{D_{i Kl}} = \text{Amount of item } i \text{ processed at plant } j \text{ and shipped to distribution centre } k \text{ in period } l.
\]

\[
DC_{i k l} = \text{Amount of item } i \text{ shipped from distribution centre } k \text{ to customer } m \text{ in period } l.
\]

\[
Y_{i k l} = \text{Inventory level for item } i \text{ at distribution centre } k \text{ at the end of period } l.
\]

\[
P_{r i} = \text{Amount of resource } r \text{ at plant } j \text{ purchased (and delivered) at the start of period } l.
\]

\[
IP_{r i} = \text{Inventory of resource } r \text{ at plant } j \text{ at the end of period } l. \text{(IP: 'inventory at plant')}.
\]

Production Constraints

\[
IP_{r i} - IP_{r i} = \sum_{i} t_{r i} \sum_{k} PD_{i k l} + IP_{r i} \quad (1)
\]

\[
P_{r i} \leq UC_{r i} \quad (2)
\]

\[
t_{r i} = \text{Amount of resource } r \text{ required to produce one unit of item } i \text{ at plant } j.
\]
\[ U_{rjl} = \text{Total usage of } r\text{th resource allowed at plant } j \text{ in period } l. \]

**Demand Constraints**

\[ \begin{align*}
\text{POTD}_{i,ml} & = D_{i,ml} \quad (l = 1) \\
\sum_k \text{DC}_{ikml} & \leq D_{i,ml} \quad (l > 1) \\
D_{i,ml} & = \text{POTD}_{i,ml} - Z_{i,ml}[D_{i,ml-1} - \sum_k \text{DC}_{ikml-1}] \\
& + (1 - Z_{i,ml})[D_{i,ml-1} - \sum_k \text{DC}_{ikml-1}] \\
& = \text{POTD}_{i,ml} + (1 - 2Z_{i,ml})[D_{i,ml-1} - \sum_k \text{DC}_{ikml-1}] \quad (5)
\end{align*} \]

\[ \text{POTD}_{i,ml} = \text{Potential demand of customer } m \text{ for item } i \text{ in period } l. \]

\[ D_{i,ml} = \text{Effective demand of customer } m \text{ for item } i \text{ in period } l. \]

\[ Z_{i,ml} = \text{Proportion of the shortage of item } i \text{ lost as a sale to customer } m \text{ (} 0 \leq Z_{i,ml} \leq 1) \text{.} \]

Justification and discussion of the concepts of potential and effective demands are contained in the body of this article.

**Inventory Balance Constraints**

\[ \sum_j \text{PD}_{ijkl} - \sum_m \text{DC}_{ikml} + Y_{i,kl-1} = Y_{i,kl} \quad (6) \]

\[ Y_{ik0} = \text{Initial stock of item } i \text{ at distribution centre } k, \text{ specified by the manager of inventory.} \]

**Inventory Level for each Item at each Distribution Centre**

\[ I_{ikl} \leq Y_{ikl} \leq I_{ikl} \quad (7) \]

\[ I_{ikl} = \text{Maximum inventory for product } i \text{ at distribution centre } k, \text{ in period } l. \]

\[ I_{ikl} = \text{Minimum inventory for product } i \text{ at distribution centre } k, \text{ in period } l. \]

**Inventory Level for each Resource at each Plant**

\[ \text{LR}_{rjl} \leq \text{IP}_{rjl} \leq U_{rjl} \quad (8) \]

\[ UR_{rjl} \text{ and LR}_{rjl} \text{ are respectively the upper and lower levels on the inventory of resource } r, \text{ at plant } j, \text{ in period } l. \]

**Non-negativity Constraints (All variables } \geq 0)\]

**Objective: Minimisation of the required logistics budget**

\[ \begin{align*}
\text{MIN} & \sum_{r,j,l} (\text{CR}_{rjl} \text{IP}_{rjl} + \text{CIP}_{rjl} \text{IP}_{rjl}) + \sum_{i,k,l} \text{CW}_{ikl} Y_{ikl} \\
& + \sum_{i,k,m,l} \text{CTDC}_{ikml} \text{DC}_{ikml} + \sum_{i,k,l} \text{CTPD}_{ijkl} \text{PD}_{ijkl} \\
\text{CTPD}_{ijkl} & = \text{Cost to transport one unit of item } i \text{ from plant } j \text{ to distribution centre } k, \text{ in period } l.
\end{align*} \]

\[ \text{CTDC}_{ikml} = \text{Cost to transport one unit of item } i \text{ from distribution centre } k \text{ to customer } m, \text{ in period } l. \]

\[ \text{CW}_{ikl} = \text{Cost of storing inventory one unit of item } i \text{ at distribution centre } k, \text{ in period } l. \]

\[ \text{CR}_{rjl} = \text{Cost of acquiring one unit of resource } r \text{ at plant } j \text{ in period } l. \]

\[ \text{CIP}_{rjl} = \text{Cost of storing in inventory one unit of resource } r \text{ at plant } j, \text{ in period } l \text{(this includes only actual cash outflows, not opportunity costs).} \]

**Objective: Maximisation of profit**

\[ \text{MAX.} \]

\[ \begin{align*}
& \sum_{i,k,m,l} \text{PR}_{ikml} \text{DC}_{ikml} - \left[ \sum_{i,j,k,l} (\text{C}_{ij} + \text{CTPD}_{ijkl}) \text{PD}_{ijkl} \right. \\
& + \sum_{i,k,l} (\text{CW}_{ikl} + \text{CH}_{ikl}) Y_{ikl} + \sum_{i,k,m,l} \text{CTDC}_{ikml} \text{DC}_{ikml} \\
& + \sum_{i,m,l} \text{CSH}_{ikml} (D_{i,ml} - \sum_k \text{DC}_{ikml}) + \sum_{r,j,l} \text{CHIP}_{rjl} \\
& + \left. \text{CIP}_{rjl} \text{IP}_{rjl} \right]
\end{align*} \]

\[ \text{CH}_{ikl} = \text{Opportunity-cost portion of carrying a unit inventory of item } i \text{ at distribution centre } k \text{ in period } l. \]

\[ \text{CSH}_{ikml} = \text{Shortage cost per unit of unsatisfied demand of customer } m \text{ for item } i \text{ in period } l. \]

\[ \text{CP}_{rjl} = \text{Cost of producing one unit of item } i \text{ at plant } j \text{ in period } l. \]

\[ \text{PR}_{ikml} = \text{Price of item } i \text{ sent from distribution centre } k \text{ to customer } m \text{ in period } l. \]

\[ \text{CHIP}_{rjl} = \text{Opportunity-cost portion of carrying one unit inventory of resource } r \text{ at plant } j, \text{ in period } l. \]

Throughout our discussion we mean, by the "minimisation of required budget", the minimisation of all real, tangible logistics expenditures that require cash outflows. "Profit", however, besides incorporating revenues and all tangible costs, contains several penalty or opportunity costs. The intangible costs included in profit are the opportunity costs of carrying resource inventories and finished-goods inventories, and the costs of shortages.

By including in profit the penalty cost of shortages, the model can control the quantity that is out of stock, and thus limit the reduction in future sales. Inclusion of the opportunity costs of resource- and finished-goods inventories recognises that a higher "profit" (hence lower opportunity costs) can also have a customer service implication (greater lost sales). Thus, our "profit" objective permits a trade-off of the two opportunity costs of inventories with the cost of lost sales. The usual profit in accounting terms can of course be found by adding back the three latter intangible costs.

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