An integrated inventory–transportation system with modified periodic policy for multiple products

Wendy W. Qu a,2, James H. Bookbinder a,*, Paul Iyogun b

a Department of Management Sciences, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1
b School of Business and Economics, Wilfrid Laurier University, Waterloo, Ontario, Canada N2L 3C5

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Abstract

Efficient management of a distribution system requires an integrated approach towards various logistical functions. In particular, the fundamental areas of inventory control and transportation planning need to be closely coordinated. Our model deals with an inbound material-collection problem. An integrated inventory–transportation system is developed with a modified periodic-review inventory policy and a travelling-salesman component. This is a multi-item joint replenishment problem, in a stochastic setting, with simultaneous decisions made on inventory and transportation policies. We propose a heuristic decomposition method to solve the problem, minimizing the long-run total average costs (major- and minor-ordering, holding, backlogging, stopover and travel). The decomposition algorithm works by using separate calculations for inventory and routing decisions, and then coordinating them appropriately. A lower bound is constructed and computational experience is reported. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction and problem setting

With the trend towards greater synergy between suppliers and industrial customers, most manufacturing enterprises are organized as networks of manufacturing and distribution sites that purchase raw materials, transform those materials into intermediate and finished products, and distribute the finished goods to customers. Management of such networks (also referred to as "supply chains") has emerged as a major topic in operations research (Lee and Billington, 1993). Improving the efficiency of these systems requires

*Corresponding author. Tel.: (519) 888 4013; fax: (519) 746 7383; e-mail: jbookbinder@uwaterloo.ca
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2 Permanent address: AT&T Labs, NJ, USA.
striking a balance between the various logistical functions; in particular, inventory control and transportation planning need to be closely coordinated.

This paper concerns the management of one segment of a supply chain by such an integrated approach: decisions for both inventory and transportation functions are made simultaneously. We consider the specific situation of a central warehouse (where all stocks are kept) and several geographically dispersed suppliers. The central warehouse replenishes its stock by dispatching vehicles to collect the goods from those vendors, on routes which begin at the warehouse and end there. The objective is to determine a replenishment strategy (inventory control rules and routing patterns) that enables the warehouse to meet its demand at minimum long-run total cost per unit time.

Blumenfeld et al. (1987) ascribe to General Motors a similarly structured supply network. In recent years, the GM network consisted of approximately 2500 suppliers worldwide, 130 GM parts plants and 30 assembly plants. Final assembly of a typical General Motors vehicle requires that roughly 13,000 parts (varying in size and value) be fabricated and assembled. These auto parts, either purchased from outside vendors or produced by the GM parts plants, are transported to and stored at central warehouses for supply to the assembly plants. Reduction of costs (inventory plus transportation) in the logistics network is therefore very important to GM and other auto manufacturers. In a different context, similar issues are also faced by an integrated wholesaler/retailer in the supply of grocery products. Our goal in this article is thus to find effective policies for inventory replenishment and transportation route design in such networks. We note in passing that the "parts" in the GM example are referred to as "items" in the present paper.

Although there is substantial literature on inventory control and transportation management, respectively, much less is available on the combined problem. The research which does address an integrated inventory-transportation system has some common features about the network, transportation and inventory management. The network often consists of one warehouse and multiple retailers, with inventory distributed by a fleet of vehicles from the warehouse to the retailers. A simple continuous review, lot-sizing inventory policy is often adopted. For transportation, a vehicle capacity constraint is typically incorporated. The resulting inventory and transportation policies usually assign retailers to routes and obtain replenishment intervals and delivery sizes for each retailer. The objective is to minimize the sum of inventory and transportation costs over a given planning horizon. The mathematical models and their corresponding algorithms differ according to the assumptions on the planning horizon (single or multiple period or infinite horizon), demand (deterministic, stochastic), and number of items (single or multiple).

Initial studies in this area began in the mid-1980s. Federgruen and Zipkin (1984) integrate the allocation and routing problems within the same model by treating a single-period, single-item problem with random demands at the retailers. No ordering cost is considered. Their method is to first decompose the problem into two portions: inventory allocation and vehicle routing. The solution then follows by constructive and/or interchange heuristics. Yano and Gerchak (1989) analyze a direct shipment situation. They assume a fixed transportation cost per vehicle per shipment. In their single-period model, only one vendor supplies multiple items to a particular point. This greatly simplifies the transportation problem and avoids route design.

Anily and Federgruen (1990) consider a single-item, deterministic model where the objective is to minimize average transportation and inventory costs over an infinite horizon. They use a policy where a retailer can be assigned to different routes simultaneously, each route satisfying possibly a fraction of the total demand. A partition method is used to derive lower and upper bounds on total costs. It is then shown that these bounds become asymptotically tight when the number of demand points tends to infinity. Anily and Federgruen (1993) extend this work to a two-echelon distribution problem where the central warehouse keeps system stocks instead of being a mere transshipment point. Gallego and Simchi-Levi (1990) present a lower bound on long-run average cost over all inventory-routing strategies for such systems. Gallego and
Simchi-Levi stress the benefits of direct shipping, whereas Hall (1992) argues that multi-stop routing strategies can still provide large savings.

For the case of deterministic demands for several products at multiple retailers, Viswanathan and Mathur (1997) consider both inventory decisions and vehicle routes from a warehouse. They obtain a stationary nested joint replenishment policy for the situation when replenishment intervals are power-of-two multiples of a base planning period.

Bell et al. (1983), Dror and Ball (1987) and Chien et al. (1989) study the inventory-routing problem, a particular type of resource allocation problem which contains the following features: a single-period model, a unique resource to allocate, deterministic customer-dependent daily demand, and replenishment decisions where stockout is not allowed. This problem, like ours, requires decisions on the period in which each customer is replenished. However, Dror and Ball (1987) have shown how to calculate each customer's delivery size, so that issues concerning inventory policy are not considered. One may therefore say that the inventory-routing problem is an extension of the vehicle routing problem with emphasis on transportation management.

Our research differs from previous work in one or more of the following ways: we consider multiple-items, several suppliers and stochastic demand over a time horizon. We integrate the inventory and transportation decisions into one mathematical model. Our solution method assumes a modified periodic-review inventory policy at the warehouse.

We model the inventory cost as consisting of a joint fixed replenishment cost to be shared among all the items included in a given replenishment, as well as an item-dependent minor cost for including any specific item in that replenishment. Inventory holding cost at the central warehouse is incurred at a constant rate per unit time. Total backlogging is assumed at the warehouse, with a cost proportional to the total number of units short. In the transportation problem, there is a fixed cost for each stopover, plus a variable cost proportional to the travel distance. Vehicle capacity is assumed to be unlimited (subsequent research will consider vehicle capacity constraints).

Operationally, each supplier is responsible for getting items ready for pick-up. It is assumed that no shortage or delay occurs for any collection. Since the model is treated from the viewpoint of the manufacturer, charges to a supplier from holding stock are not considered.

The remainder of this paper is organized as follows. In Section 2, we formulate the model in detail. A heuristic decomposition method for solving the problem is developed in Section 3 while a lower bound is constructed in Section 4. The special case when each supplier produces only one item, and for which we will find a tighter lower bound, is also studied. Section 5 presents computational results, including the service levels attained under the modified periodic policy. Section 6 gives conclusions. (A complete listing of our notation appears in Appendix A.)

2. Model formulation

A central warehouse carries the system stock. In a particular period, a vehicle (owned or controlled by the warehouse) is dispatched from there to collect items from certain suppliers. Material replenishment coordinated in this way is an example of a "shipment consolidation" program of inbound deliveries (Higgins and Bookbinder, 1994).

The total cost incurred thus consists of the transportation cost, which includes dispatching, stopover and routing costs, and the inventory cost (ordering, holding and backlog costs). We wish to determine procedures for inventory management and vehicle routing so that the warehouse may satisfy demand at a minimum long-run average cost per unit time.

Since the structure of the optimal solution is unknown, our work restricts attention to a class of inventory policies and finds an optimum within the selected class, namely a modification of the periodic
review \((R_i, T_i)\) policy. In the usual form of an \((R_i, T_i)\) policy, item \(i\) is replenished up to \(R_i\) every \(T_i\) periods. However, in the modified periodic inventory policy (MP), each \(T_i\) is an integer multiple of a base period \(T\). \(T\) corresponds to the order cycle of a set of items, the so called "base items" that are re-stocked most frequently. In our model, we will choose the base period \(T\) by optimization.

Because the review intervals \(T_i\) may differ, a varying number of items will be jointly ordered in each period. The items replenished in a given period \(j\) consist of the base items, plus all items \(i\) for which \(jT\) is an integer multiple of \(T_i\). Consider three items with \(T_1 = 1\), \(T_2 = 2\) and \(T_3 = 3\) time units respectively; the base period is thus \(T = 1\). Assume (here and throughout) that the cycle begins at period 0. All three items will then be ordered in periods 0, 6, 12, etc. In periods 2, 4, 8, 10, etc. only items \(i = 1\) and \(2\) will be replenished, while for periods \(j = 3\), \(9\), \(15\), etc., just items \(i = 1\) and \(3\) will be re-stocked. In this example, there is only the single base-item \(i = 1\). It will be ordered in every period (since \(T = 1\)), sometimes jointly with other items, but by itself in periods \(1, 5, 7, 11, 13\), etc.

Under the MP policy, inventory replenishment is a regeneration process, with the length of each cycle \((MT)\) as the least common multiple of all \(T_i\) \((M = 6\) periods in the preceding example). Since the expected behaviour in each cycle is the same, the long-run cost per unit time is equal to the mean cost over one cycle.

The expected total cost in a regeneration cycle \(MT\) is made up of transportation, ordering, holding and backlogging costs. For each replenishment period \(j\), the transportation cost includes the costs of stopover (at those plants visited) plus the travelling costs. Over a regeneration cycle,

\[
\text{Average Transportation Cost} = \frac{\sum_{j=1}^{M} \left( \sum_{p=1}^{n} y_{jp} L + cD(S_j) \right)}{MT}.
\]

The ordering cost consists of a joint fixed cost \((K)\) for placing an order, and minor ordering costs \((k_{ij})\) for each item \(i\) to be included in that order. Hence,

\[
\text{Average Ordering Cost} = \frac{\sum_{j=1}^{M} \left( K + \sum_{i=1}^{N} y_{ij} k_{ij} \right)}{MT} = K/T + \text{MOC},
\]

where the average minor ordering cost is

\[
\text{MOC} = \frac{\sum_{i=1}^{K} k_{ii}}{m_{i}T}.
\]

As presented in Hadley and Whitin (1963), it is convenient to calculate the holding cost between the arrival of two successive orders, rather than between the placement of those orders. When an order is placed, the warehouse waits a constant lead time \(L\) for the items to arrive, after which the expected inventory level for item \(i\) is \(R_i - \lambda_i L\). The next order arrives after \(m_i T + L\), and the corresponding inventory level just before arrival of that order is \(R_i - \lambda_i (m_i T + L)\). The average net inventory level (on-hand minus backorders) of item \(i\) during the interval is thus approximately \(R_i - \lambda_i (L + m_i T/2)\). To that degree of accuracy, the mean long-run holding cost is

\[
H = \sum_{i=1}^{N} h_i [R_i - \lambda_i (L + m_i T/2)].
\]  \hspace{1cm} (1)

A stockout occurs following an order placed at time \(t\) if the cumulative demand \(x_t\) (between \(t\) and \(t + L + m_i T\)) exceeds \(R_i\). There will then be a penalty cost \(p_i\) for each unit backlogged. If demand has a density function \(f(x_t, L + m_i T)\) over the interval of length \(L + m_i T\), the mean backlogging cost in the long-run is
\[ BL = \sum_{i=1}^{N} \left( \frac{\pi_i}{m_i T} \int_{x_i}^{\infty} (x_i - R_i) f(x_i, L + m_i T) \, dx_i \right). \]  
\[ (2) \]

We assume that demands for given items are independent and identically distributed in the form of a Brownian motion process. That is, when we fix the decision variables \( m_i \) and \( T \), the demand over any particular time interval of length \( \tau \) is normally distributed, with \( E(x_i, \tau) = \lambda_i \tau \) and \( \text{Var}(x_i, \tau) = \delta_i \tau \).

Let \( G = MOC + H + BL \). Then the objective is to minimize the total long-run expected cost \( C \) in problem (P).

\[(P) \quad \min \ C = \sum_{j=1}^{M} \left( \sum_{p=1}^{p} \gamma_{jp} k_{jp} + cD(S_j) \right) / MT + K/T + G, \]
\[ (3) \]

\[ s.t. \]
\[ M = \text{Least Common Multiple of } (m_1, m_2, \ldots, m_N), \]
\[ (4) \]
\[ \gamma_{ij} = \begin{cases} 1 & \text{if } m_i \text{ divides } j \text{ exactly,} \\ 0 & \text{otherwise,} \end{cases} \]
\[ (5) \]
\[ \gamma_{ji} = \begin{cases} 1 & \text{if } \sum_{i=1}^{N} \gamma_{ip} \theta_{ip} > 0, \\ 0 & \text{otherwise,} \end{cases} \]
\[ (6) \]
\[ \emptyset \neq S_j \subseteq S, \]
\[ (7) \]
\[ m_i \text{ is integer; } R_i, T \geq 0 \text{ and } 1 \leq i \leq N, 1 \leq j \leq M \text{ and } 0 \leq p \leq P. \]
\[ (8) \]

If item \( i \) is produced by supplier \( p \) (\( \theta_{ip} = 1 \)) and replenished in period \( j \) (\( \gamma_{ij} = 1 \)), then obviously supplier \( p \) is visited in period \( j \) (\( \gamma_{ji} = 1 \)). It is over this subset of suppliers that a vehicle route must be constructed.

Problem (P) has the interesting feature that \( M \), the number of periods, is a decision variable rather than a given constant. The route of minimal distance \( D(S_j) \), connecting those suppliers \( S_j \) visited in period \( j \), is obtained by solving a travelling-salesman problem. \( M \) such problems are nested in the original problem, so that we solve a TSP for each period \( 1 \leq j \leq M \).

3. Solution method

The approach in this research is to decompose the model into two parts: an inventory problem and a transportation problem. The overall model is solved by iterating between these two problems. Observe that if the joint costs are excluded from Eq. (3), \( C \) will be additive in items because of the separability of \( G \). Decomposition of total cost \( C \) by item can thus be achieved by fixing transportation (travel and stopover) cost, and then allocating it, together with joint ordering cost, to each item in every period. With \( F_i \) as this allocated transportation cost, the result is a conventional inventory problem (IP):

\[(IP) \quad \min \ C_i = \sum_{i=1}^{N} F_i/m_j T + K/T + G, \]
\[ (9) \]

\[ \text{s.t. } m_i \text{ integer; } R_i \text{ and } T \geq 0, \text{ and } 1 \leq i \leq N. \]

We will show below that the fixed transportation costs in each period can be obtained through solving an independent transportation subproblem. Thus, the model is decomposed into two portions: inventory (master problem) and transportation (subproblem), with the former solved item by item and the latter period by period. Solutions to the decomposed inventory and vehicle routing models can be found in a straightforward manner.
3.1. Inventory master problem

We have defined $F_i$ as the shared transportation cost (travel plus stopover) for each item. $F_i$ is iteratively updated with reference to the transportation subproblem (Section 3.2), but its most recent value is a known parameter in IP. The $F_i$-portion is thus separable by item, as is the portion involving $G$.

The $K/T$-term of Eq. (9) can also be made separable in items through a further decomposition. This is obtained by allocating the joint ordering cost $K$ according to a scheme suggested by Atkins and Iyigun (1987, 1988). Their method partitions the items into two groups: base items and nonbase items. The base items $B$ are those for which $m_i = 1$, hence are replenished every time any of the $N$ items is replenished. $K$ is allocated to base items only. We denote by $B'$ the set of nonbase items for which $m_i > 1$.

Partitioning items into these two groups is done by solving two problems, (IP1) and (IP2), for given values of the $F_i$. The first problem is

\begin{equation}
(IP1) \quad \min \quad C_i = \frac{(F_i + k_{ii})}{T_i} + h_i[R_i - \lambda_i(L + \bar{T}_i/2)] + \left(\frac{\pi_i}{T_i}\right) \int_{x_i}^{\infty} (x_i - R_i) f(x_i, L + \bar{T}_i) \, dx_i,
\end{equation}

s.t. $R_i, \bar{T}_i \geq 0$.

In (IP1), we obtain $\bar{T}_i, \bar{T}_i$ need not be an integral multiple of $T$. $\bar{T}_i$ should thus not be confused with $T_i$, which will be calculated as $T_i = m_i T$ once we have obtained $m_i$ below.

We rank in ascending order the $\bar{T}_i$ found from Eq. (10), select the item (or items) with the smallest $\bar{T}_i$, and let that item (or those) be in $B$. We then obtain $T$ for the one or more base items which thus share $K$, and which now have $m_i = 1$, from problem (IP2)

\begin{equation}
(IP2) \quad \min \quad C_B = K/T + \sum_{i \in B} \left\{ \frac{(F_i + k_{ii})}{T} + h_i[R_i - \lambda_i(L + T/2)] \right. \\
\left. + \left(\frac{\pi_i}{T}\right) \int_{x_i}^{\infty} (x_i - R_i) f(x_i, L + T) \, dx_i \right\}
\end{equation}

with $R_i, T \geq 0$. (IP2) is a single-period problem, and can be solved directly to give us the base period $T$ and the order-up-to level $R_i$.

We then compare the next smallest $\bar{T}_i$ from (IP1) with $T$ from (IP2). If, $\bar{T}_i \leq T$, let this item belong to $B$, and solve (IP2) to update $T$. Continue until, for each item $i$ not in $B$, $\bar{T}_i > T$. These are nonbase items. This partitioning process will yield the two groups of items and their corresponding replenishment cycles.

For items in group $B'$, we round $\bar{T}_i/T$ to the nearest integer $m_i$ (rounding up or down as appropriate). More precisely, defining $u \equiv \bar{T}_i/T$, the structure of the cost function suggests that we round $u$ to $m_i$ when $m_i(m_i - 1) < u^2 \leq m_i(m_i + 1)$. The integer $m_i$ enables us to calculate $T_i$. The value $m_i$ is also employed in the transportation subproblem.

3.2. Transportation subproblem

Given the $m_i$ from (IP1) and (IP2), we know which items will be replenished ($y_{ij}$) and the particular suppliers visited in a given period $j$ ($y_{ij}$). We must then design a minimum-cost transportation route to those suppliers. With no constraint on vehicle capacity, this is a typical TSP problem. Since for each period the vehicle routing model is an independent problem, we can use any TSP approach, such as the travelling-salesman model of Gavish and Graves (1978).
The result provides the optimal route, and the distance travelled \( D(S_j) \), in each period. Given the transportation cost per unit distance, we can allocate by a heuristic method the total travel cost per period to the corresponding items collected in that period. That travel cost \( cD(S_j) \) is first apportioned to each supplier based on its distance from the central warehouse. This is because the further that plant is located from the warehouse, the higher would be the transportation cost if the plant were visited individually. This distance should also affect replenishment cycles \( T_i \) of items collected from that supplier. In order to lower transportation cost per unit time, items produced by far-away suppliers will tend to have longer replenishment cycles.

Detailed calculation of \( F_i \), the shared transportation cost in a replenishment cycle, is contained in Appendix B. For given \( m_i \) and \( D(S_j) \), we have

\[
F_i = (m_i/M) \sum_{j=1}^{N} \left[ \left( \frac{\gamma_j \theta_j}{\sum_{j=1}^{N} \gamma_j \theta_j} \right) \left( \frac{A_{ij} d_{ij}^2}{\sum_{j=1}^{N} \gamma_j d_{ij}} \right) cD(S_j) + k_{ip} \right].
\]

(12)

\( F_i \) is then used in the inventory master problem.

3.3. Summary of the solution procedure

The overall sequence of solution is thus:

**Step 1**: For each item \( i \): Initialize \( m_i = 0 \) and \( F_i = 2c \sum_{p=1}^{p} d_{ip} \theta_{ip}/\sum_{p=1}^{p} \theta_{ip} \nu(p) \), where \( \nu(p) = \sum_{i=1}^{N} \theta_{ip} \) is the number of products supplied by location \( p \).

**Step 2**: Substitute \( F_i \) into (IP1) and solve for \( (R_i, T_i) \) for each item.

**Step 3**: Relabel products in nondecreasing order, with the item of smallest \( T_i \) as item [1] and the largest as item [N]. Now item [1] is a base item. Solve (IP2) for \( T \). Compare \( T \) with \( \tilde{T}_{[1]} \); if \( T \geq \tilde{T}_{[1]} \), include item [2] in \( B \), and solve (IP2) again. Continue until \( T < \tilde{T}_{[i+1]} \). This occurs for \( i = i^* \), say. The final value of \( T \) is then the replenishment interval for base items \( i = [1], \ldots, [i^*] \). The \( T_i \) of nonbase items \( i = [i^*+1], \ldots, [N] \) remain the same from step 2.

**Step 4**: Check that each \( T_i \) is an integer multiple of \( T \). If not, adjust \( T_i \) to \( T_i = m_i T \), appropriately rounding to the nearest integer \( m_i = \lceil T_i/T \rceil \) as discussed above. With \( T_i \) replacing \( \tilde{T}_i \) in Eq. (10), calculate an updated \( R_i \) from that equation. Compare the new \( m_i \) to the previous values. If they are the same, STOP. Otherwise go to step 5.

**Step 5**: Given \( m_i \) and hence \( S_j \), solve the vehicle routing problems for each \( j = 1, 2, \ldots, M \), obtaining the \( D(S_j) \).

**Step 6**: Allocate total transportation cost to each item, calculating the share \( F_i \) from Eq. (12). Go to step 2.

4. Lower bound

The cost from the above heuristic method constitutes an upper bound to the optimal solution. Since the minimum cost for problem (P) is unknown, a lower bound must be found to test the effectiveness of the heuristic. This lower bound can be derived as follows. We first allocate joint costs \( (K, cD(S_j) \) and \( k_{ip} \) to each item in such a way that the sum of the allocated costs over items is less than or equal to the original joint costs. The sum of \( G \) plus these single-item costs provides a lower bound for the entire coordinated multi-item system. Such a lower-bound cost function has been designed to be separable in items so that it can be minimized one item at a time.
Let us denote by \( C_L \) the best lower bound in the set constructed by the above method. \( C_L \), obtained via a joint-cost allocation scheme proposed by Atkins and Iyogun (1987), is proved to provide the maximum lower bound in that set (Iyogun and Atkins, 1993). For simplicity, we use a single weight \( \alpha_i \) for all the joint costs of any period \( j \), where

\[
\alpha_i \geq 0 \text{ for all } i \in N, \quad \text{and } \sum_{i=1}^{N} \alpha_i = 1. \tag{13}
\]

Let us define \( \tilde{G} = \text{MOC} + H + \text{BL} \) when \( \tilde{T} \) replaces \( m_i T \) in the cost terms. Following a preliminary lemma, we will show that \( C_L \) is the desired lower bound for \( C \), where

\[
C_L = \max_{\alpha} \left\{ \min_{R_i, \tilde{T}_i} \left\{ \sum_{i=1}^{N} \alpha_i \left[ K + \sum_{p=1}^{P} \theta_{ip} (2c_{dop} / P + k_{ip}) \right] + \tilde{G} \right\} \right\} \tag{14}
\]

In more detail, for any period \( j \), \( \alpha_i \) is the fraction of the total joint costs that will be allocated to item \( i \). Naturally, as in Eq. (13), the weights \( \alpha_i \) are nonnegative and sum to 100%. The optimal lower bound, i.e., the greatest lower bound available within this cost-allocation scheme, was proved by Iyogun and Atkins (1993) to result from solving the maximization problem (14). There, the weights \( \alpha_i \) are decision variables that play the role of "multipliers" (in the objective function) of the joint costs that are to be allocated.

**Lemma 1.**

\[
D(S_j) \geq 2y_{ij} \left( \sum_{p=1}^{P} \theta_{ip} d_{op} \right) / P. \tag{15}
\]

**Proof.** The triangle inequality for the travel distance gives

\[
y_{ij} D(S_j) \geq 2y_{ip} d_{op}. \tag{16}
\]

Summing Eq. (16) over all suppliers, and using \( P \geq \sum_{p=1}^{P} y_{ip} \) and \( y_{ij} \geq y_{ij} \theta_{ip} \), yields

\[
D(S_j) \geq \left( \sum_{p=1}^{P} y_{ip} 2d_{op} \right) / \left( \sum_{p=1}^{P} y_{ip} \right) \geq \left( \sum_{p=1}^{P} y_{ip} 2d_{op} \right) / P \geq 2y_{ij} \left( \sum_{p=1}^{P} \theta_{ip} d_{op} \right) / P. \quad \square
\]

**Lemma 2.**

\( C_L \) is a lower bound on \( C \).

**Proof.** Because of Eq. (13), with \( y_{ij} = \{0, 1\} \),

\[
\sum_{i=1}^{N} \alpha_i y_{ij} \leq 1. \tag{17}
\]

From Lemma 1, Eq. (17) and \( y_{ip} \geq y_{ij} \theta_{ip} \), it can be easily derived that

\[
K + cD(S_j) + \sum_{p=1}^{P} \gamma_{ip} k_{ip} \geq \sum_{i=1}^{N} \alpha_i y_{ij} \left[ K + \sum_{p=1}^{P} \theta_{ip} (2c_{dop} / P + k_{ip}) \right]. \tag{18}
\]
Substituting Eq. (18) into Eq. (3),

\[
C = \min_{R, T, m_i} \left\{ \sum_{j=1}^{M} \left[ \sum_{p=1}^{P} \gamma_{jp} k_{tp} + cD(j) \right] / MT + K/T + G \right\} \text{s.t.: (4)-(8)}
\]

\[\geq \max_{a_{ij}} \left\{ \min_{R, T, m_i} \left\{ \sum_{i=1}^{N} a_i \left[ K + \sum_{p=1}^{P} \theta_{ip}(2cd_{op}/P + k_{ip}) \right] / m_i T + G \right\} \right\} \text{s.t.: (4), (5), (8) and (13)} \}

\[= \max_{a_{ij}} \left\{ \min_{R, T, m_i} \left\{ \sum_{i=1}^{N} a_i \left[ K + \sum_{p=1}^{P} \theta_{ip}(2cd_{op}/P + k_{ip}) \right] / m_i T + G \right\} \right\} \text{s.t.: (13); } m_i \text{ as integers; } R, T \geq 0 \}

\]

Later, in discussing our computations, we shall always mean this maximal lower bound \(C_L\) when we refer to "lower bound". (See Tables 5, 6 and 9 below.)

4.1. Procedure for selecting \(a_i\)

In light of the above, we obtain \(a_i\) as follows:

Step 1: Define FF = \(K + \sum_{p=1}^{P} \theta_{ip}(2cd_{op}/P + k_{ip})\). Set \(a_i = 0\) in Eq. (14), and obtain the optimal \((R_i, \tilde{T}_i)\) for each item by minimizing \(G\).

Step 2: Relabel products in nondecreasing order, with the item of smallest \(\tilde{T}_i\) as item [1] and the largest as item [N]. Now solve Eq. (14) for \(a_i \geq 0\). Allocate a fraction \(q_{i[1]}\) FF to item [1] and solve Eq. (14) again until items [1] and [2] have identical replenishment periods \(\tilde{T}_i\). Continue to allocate joint costs to each in the correct proportions \(q_{i[1]}, a_{i[2]}\), increasing both of their replenishment periods until they coincide with that of item [3]. Continue until the entire cost FF has been allocated, i.e. \(\sum_{i=1}^{N} a_i = 1\), thereby obtaining the set of \(a_i\) and the corresponding total costs for the lower bound.

4.2. A special case

In the preceding model, each supplier produces one or more items (parts). In the special case when each supplier produces exactly one item, \(\theta_{ip}\) will be the identity matrix, hence \(\gamma_{ip} = y_{ip}\). Stopover cost \(k_{ip}\) is specific to the item \(i\) produced by \(p\). For lower bound construction in this case, Eq. (16) becomes \(y_{ip}[D(S_j) - 2d_{ao}] \geq 0\), an inequality tighter than Eq. (15). As a result, following the method of proof of Lemma 2, a better lower bound \(C_L^*\) can be obtained

\[
C_L^* = \max_{a_{ij}} \left\{ \min_{R, \tilde{T}_i} \left\{ \sum_{i=1}^{N} a_i (K + 2cd_{oi}) / \tilde{T}_i + G \right\} \right\} \text{s.t.: (13); } R, \tilde{T}_i \geq 0 \}
\]

(19)
5. Computational experience

5.1. Initial tests

In our initial computational tests, the model consists of one central warehouse, three suppliers and four items \(1 \leq i \leq 4\), with supplier \(p = 3\) producing both items 3 and 4. For the geographical layout and input data, see Tables 1–3. That input was generated from information in Example 5.2 of Hadley and Whitin (1963).

Table 4 lists the values of some important decision variables and parameters at each iteration: the values \((F_i, m_i)\) transferred between the master problem and subproblem during the iteration process; the decision variables \(\bar{T}_i\) and \(R_i\); and the total cost \(C_i\).

We found that the differences between the total cost \(C\) in Eq. (3) and \(C_i\) of the inventory master problem (9) are zero during the entire iteration process. By substituting Eq. (12) into Eq. (9) and recalling that \(T_i = m_i\bar{T}\), it is straightforward to show theoretically that Eq. (3) and Eq. (9) are identical for all \((R_i, T_i)\).

The solutions of Table 4 converged after two iterations. Our computational experience with several other tests also indicates that with properly chosen initial values of \(F_i\), the solution process converges within a small number of iterations.

It turns out that even if the entire cost \(K\) were allocated to item 3, its lengthened replenishment cycle would still remain below \(\bar{T}_2\). Thus, before adjusting the \(\bar{T}_i\) for consolidation, item 3 is the only base item (Table 4). Post-adjustment results show that items 2, 4 join item 3 as base items.

In this computation, the total cost increase due to adjusting \(\bar{T}_i\) for consolidation is only 0.3%. Such a small value result because the allocation scheme for \(K\) is designed to lessen the differences in replenishment cycles \(\bar{T}_i\). Since more items become base items and fewer \(\bar{T}_i\) need be adjusted, there is minimal final cost increase due to the adjustment process.

In Table 5, we construct the maximal lower bound as in Section 4; the same \(z_i\) is employed for all three joint costs \((K, cD(S)\) and \(k_{\text{mp}})\). With lower bound as 7883, the recommended heuristic solution of Table 4 is within 14.4% of that lower bound, indicating satisfactory performance by our algorithm (Section 3.3) in solving the problem.

The cost/LB ratios in Table 6 demonstrate the effectiveness of the heuristic method under various \(K\). During this computation, we found that the cost increase due to adjustment for consolidation was insubstantial for each \(K\). In addition, the cost/LB ratios are satisfactory in every case. Finally, these ratios improve as \(K\) increases. That indicates our heuristic method performs well overall, and especially when \(K\) is large.

5.2. Service levels

It is important to check the service levels attained under the modified periodic inventory policy. We employ the two most commonly used measures: \(\Phi\), is the probability of no stockout in a replenishment

<table>
<thead>
<tr>
<th>Table 1: Input data</th>
<th>Item 1</th>
<th>Item 2</th>
<th>Item 3</th>
<th>Item 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\lambda) (units/year)</td>
<td>600</td>
<td>900</td>
<td>1200</td>
<td>1000</td>
</tr>
<tr>
<td>(\delta) (units/year)</td>
<td>800</td>
<td>600</td>
<td>700</td>
<td>500</td>
</tr>
<tr>
<td>(L) (years)</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>(h) ($/unit/year)</td>
<td>5.6</td>
<td>21</td>
<td>42</td>
<td>15</td>
</tr>
<tr>
<td>(\pi) ($/unit)</td>
<td>28</td>
<td>35</td>
<td>40</td>
<td>30</td>
</tr>
<tr>
<td>(k_p) ($/order)</td>
<td>25</td>
<td>14</td>
<td>20</td>
<td>30</td>
</tr>
</tbody>
</table>

The \(T_i\), resulting from the model (in years) is converted to days based on 1 year = 365 days. Two additional constants: \(c = 0.5\) $/mile and \(K = $100\) per order.
Table 2
Geographical layout (distances in miles)

<table>
<thead>
<tr>
<th>Warehouse</th>
<th>Supplier 1</th>
<th>Supplier 2</th>
<th>Supplier 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>11</td>
<td>9</td>
<td>7</td>
</tr>
<tr>
<td>Supplier 1</td>
<td>11</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>Supplier 2</td>
<td>9</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>Supplier 3</td>
<td>7</td>
<td>8</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 3
Items produced by suppliers (θᵢᵢ)

<table>
<thead>
<tr>
<th>Item</th>
<th>Supplier 1</th>
<th>Supplier 2</th>
<th>Supplier 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>kᵢᵢ (#/stop)</td>
<td>40</td>
<td>50</td>
<td>60</td>
</tr>
</tbody>
</table>

Table 4
Sequence of solutions in the computational tests

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Items</th>
<th>Fᵢ</th>
<th>Tᵢ</th>
<th>mᵢ</th>
<th>Rᵢ</th>
<th>Cᵢ = C</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Item 1</td>
<td>11.0</td>
<td>49.3</td>
<td>2</td>
<td>114.6</td>
<td>7312.0</td>
</tr>
<tr>
<td></td>
<td>Item 2</td>
<td>9.0</td>
<td>23.0</td>
<td>1</td>
<td>87.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Item 3</td>
<td>3.5</td>
<td>23.0</td>
<td>1</td>
<td>111.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Item 4</td>
<td>3.5</td>
<td>23.0</td>
<td>1</td>
<td>95.0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Item 1</td>
<td>45.9</td>
<td>70.5</td>
<td>3</td>
<td>151.0</td>
<td>9087.7</td>
</tr>
<tr>
<td></td>
<td>Item 2</td>
<td>56.1</td>
<td>30.3</td>
<td>1</td>
<td>105.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Item 3</td>
<td>36.4</td>
<td>28.1</td>
<td>1</td>
<td>127.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Item 4</td>
<td>36.4</td>
<td>33.2</td>
<td>1</td>
<td>123.4</td>
<td></td>
</tr>
<tr>
<td>2 (final, pre-adjusted)</td>
<td>Item 1</td>
<td>45.9</td>
<td>70.5</td>
<td>3</td>
<td>151.0</td>
<td>8999.5</td>
</tr>
<tr>
<td></td>
<td>Item 2</td>
<td>56.5</td>
<td>30.3</td>
<td>1</td>
<td>105.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Item 3</td>
<td>32.5</td>
<td>27.7</td>
<td>1</td>
<td>126.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Item 4</td>
<td>32.5</td>
<td>32.1</td>
<td>1</td>
<td>120.6</td>
<td></td>
</tr>
<tr>
<td>3 (recommended, after adjustment of Tᵢ to final Tᵢ)</td>
<td>Item 1</td>
<td>45.9</td>
<td>83.2</td>
<td>3</td>
<td>172.6</td>
<td>9021.2</td>
</tr>
<tr>
<td></td>
<td>Item 2</td>
<td>56.5</td>
<td>27.7</td>
<td>1</td>
<td>99.3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Item 3</td>
<td>32.5</td>
<td>27.7</td>
<td>1</td>
<td>126.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Item 4</td>
<td>32.5</td>
<td>27.7</td>
<td>1</td>
<td>108.3</td>
<td></td>
</tr>
</tbody>
</table>

The values Fᵢ in row 0 were furnished as inputs to the master problem. K = 100.

Table 5
Lower bound

<table>
<thead>
<tr>
<th>Item 1</th>
<th>Item 2</th>
<th>Item 3</th>
<th>Item 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>αᵢ</td>
<td>Tᵢ</td>
<td>Rᵢ</td>
<td>Cᵢ</td>
</tr>
<tr>
<td>0.0</td>
<td>40.9</td>
<td>99.6</td>
<td>7883.1</td>
</tr>
<tr>
<td>0.245</td>
<td>25.6</td>
<td>94.1</td>
<td></td>
</tr>
<tr>
<td>0.690</td>
<td>25.6</td>
<td>120.1</td>
<td></td>
</tr>
<tr>
<td>0.065</td>
<td>25.6</td>
<td>102.7</td>
<td></td>
</tr>
</tbody>
</table>

αᵢ is selected as in Section 4, to obtain the maximum lower bound Cᵢ (Eq. (14)). K = 100.
Table 6
Sensitivity study on joint ordering cost (total costs for the data of Tables 1–3)

<table>
<thead>
<tr>
<th></th>
<th>( K = 100 )</th>
<th>( K = 300 )</th>
<th>( K = 500 )</th>
<th>( K = 700 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_1 )</td>
<td>8990.5</td>
<td>11239.6</td>
<td>12968.6</td>
<td>14384.4</td>
</tr>
<tr>
<td>( C_2 )</td>
<td>9021.2</td>
<td>11242.7</td>
<td>12993.3</td>
<td>14455.8</td>
</tr>
<tr>
<td>( C_L )</td>
<td>7883.1</td>
<td>10262.6</td>
<td>12105.7</td>
<td>13672.1</td>
</tr>
<tr>
<td>( C_2/C_L ) (%)</td>
<td>114.4</td>
<td>109.6</td>
<td>107.3</td>
<td>105.2</td>
</tr>
</tbody>
</table>

\( C_1 \) denotes pre-adjusted total cost, while \( C_2 \) is the total cost after adjustment. \( C_L \) is the maximum lower bound (14) on total cost. The value \( K = 100 \) pertains to Tables 4 and 5.

cycle; \( \mathcal{P}_2 \), the fill rate, is the fraction of demand satisfied routinely from shelf (without backlog). Formulae for \( \mathcal{P}_1 \) and \( \mathcal{P}_2 \) are adapted from Schneider (1981):

\[
\mathcal{P}_1 = F(R_0, L + T_1),
\]

\[
\mathcal{P}_2 = 1 - \left( \frac{1}{\lambda T_1} \right) \int_{0}^{\infty} (x_t - R_0)f(x_t, L + T_1) \, dx_t - \int_{R_1}^{\infty} (x_t - R_0)f(x_t, L) \, dx_t.
\]

Results listed in Table 7 indicate high values of both service measures, particularly for fill rate \( \mathcal{P}_2 \). The service levels were tabulated as a check on the overall reasonableness of parameters such as \( \pi_i \). With those values, our model and its heuristic solutions lead to policies whose service levels are not unacceptably low.

We also notice that when \( K \) increases, \( \mathcal{P}_1 \) decreases while \( \mathcal{P}_2 \) remains the same. If one were going to modify our model by eliminating \( \pi_i \) from the cost functions, but adding a service-level constraint, the logical measure would thus be \( \mathcal{P}_1 \), because it is more sensitive to changes in the major ordering cost, \( K \).

5.3. Large scale problems

The heuristic method and the lower bound were tested on several groups of large scale problems varying in size and in some of the input parameters. Each case has seven suppliers.

The large scale problems are divided into four groups, based on the problem sizes of 15–50 items. Within each group, two tests are carried out with major ordering cost \( K \) equal to 100 and 500, respectively. All these problems are solved by following the procedure above. A typical computational result is given in Table 8; Table 9 summarizes all of these tests.

Let us first fix \( K \) and compare different problem sizes, i.e. Tests 1, 3, 5, 7 and Tests 2, 4, 6, 8. We observe that as the number of items relative to the number of suppliers goes up, the average \( T_i \) decreases. When a greater number of items can share the joint costs, these items can thus afford to be replenished more frequently. Under the same condition, \( C/LB \) also decreases.

Later tests indicated that if we keep the number of items the same and increase the number of suppliers, \( C/LB \) increases. This shows that it is the number of items per supplier, rather than only the number of

Table 7
Service levels attained (\%)

<table>
<thead>
<tr>
<th></th>
<th>Item 1</th>
<th>Item 2</th>
<th>Item 3</th>
<th>Item 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K = 100 )</td>
<td>( \mathcal{P}_1 )</td>
<td>95.44</td>
<td>95.44</td>
<td>92.02</td>
</tr>
<tr>
<td></td>
<td>( \mathcal{P}_2 )</td>
<td>99.81</td>
<td>99.79</td>
<td>99.68</td>
</tr>
<tr>
<td>( K = 300 )</td>
<td>( \mathcal{P}_1 )</td>
<td>95.76</td>
<td>93.64</td>
<td>88.87</td>
</tr>
<tr>
<td></td>
<td>( \mathcal{P}_2 )</td>
<td>99.82</td>
<td>99.75</td>
<td>99.60</td>
</tr>
</tbody>
</table>
Table 8
Solution for test 1 (15 items, 7 suppliers, $K = 100$)

<table>
<thead>
<tr>
<th></th>
<th>$P_i$</th>
<th>$T_i$</th>
<th>$R_i$</th>
<th>$C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item 1</td>
<td>12.8</td>
<td>52.9</td>
<td>146.5</td>
<td>21953.4</td>
</tr>
<tr>
<td>Item 2</td>
<td>19.0</td>
<td>26.4</td>
<td>163.6</td>
<td></td>
</tr>
<tr>
<td>Item 3</td>
<td>31.0</td>
<td>26.4</td>
<td>160.4</td>
<td></td>
</tr>
<tr>
<td>Item 4</td>
<td>31.0</td>
<td>26.4</td>
<td>127.9</td>
<td></td>
</tr>
<tr>
<td>Item 5</td>
<td>45.6</td>
<td>52.9</td>
<td>171.4</td>
<td></td>
</tr>
<tr>
<td>Item 6</td>
<td>49.0</td>
<td>26.4</td>
<td>99.7</td>
<td></td>
</tr>
<tr>
<td>Item 7</td>
<td>30.0</td>
<td>26.4</td>
<td>161.1</td>
<td></td>
</tr>
<tr>
<td>Item 8</td>
<td>23.1</td>
<td>26.4</td>
<td>98.8</td>
<td></td>
</tr>
<tr>
<td>Item 9</td>
<td>13.9</td>
<td>26.4</td>
<td>146.6</td>
<td></td>
</tr>
<tr>
<td>Item 10</td>
<td>13.9</td>
<td>26.4</td>
<td>148.3</td>
<td></td>
</tr>
<tr>
<td>Item 11</td>
<td>11.1</td>
<td>26.4</td>
<td>134.6</td>
<td></td>
</tr>
<tr>
<td>Item 12</td>
<td>12.0</td>
<td>26.4</td>
<td>108.3</td>
<td></td>
</tr>
<tr>
<td>Item 13</td>
<td>12.0</td>
<td>26.4</td>
<td>69.8</td>
<td></td>
</tr>
<tr>
<td>Item 14</td>
<td>31.3</td>
<td>52.9</td>
<td>184.8</td>
<td></td>
</tr>
<tr>
<td>Item 15</td>
<td>32.2</td>
<td>52.9</td>
<td>251.1</td>
<td></td>
</tr>
</tbody>
</table>

Table 9
Cost comparison for large scale problems (7 suppliers)

<table>
<thead>
<tr>
<th></th>
<th>Group 1</th>
<th>Group 2</th>
<th>Group 3</th>
<th>Group 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Test 1</td>
<td>Test 2</td>
<td>Test 3</td>
<td>Test 4</td>
</tr>
<tr>
<td># of items</td>
<td>15</td>
<td>15</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>Major K</td>
<td>100</td>
<td>500</td>
<td>100</td>
<td>500</td>
</tr>
<tr>
<td>Avg. $T_i$</td>
<td>33.5</td>
<td>36.2</td>
<td>27.9</td>
<td>28.8</td>
</tr>
<tr>
<td>Total Cost $C$</td>
<td>21953</td>
<td>23950</td>
<td>33726</td>
<td>38892</td>
</tr>
<tr>
<td>LB</td>
<td>16914</td>
<td>22199</td>
<td>27965</td>
<td>34654</td>
</tr>
<tr>
<td>C/LB(%)</td>
<td>129.8</td>
<td>116.9</td>
<td>120.6</td>
<td>112.2</td>
</tr>
</tbody>
</table>

items, that matters. The lower bound is tighter when more suppliers are visited each period. This condition is more likely met, with a larger item/supplier ratio. Hence, the ratio C/LB improves with a greater number of items relative to the number of suppliers.

Comparing test results in the same group (same problem size yet different $K$), our observation from Table 9 is quite similar to that for Table 6. The average $T_i$ and total cost increase due to higher $K$, yet C/LB decreases. That is, the method performs better as $K$ goes up.

From our computational experience, we conclude that the heuristic method and the lower bound are more effective when both the joint fixed cost, and the ratio of items per supplier, are large. Thus, with a given number of suppliers, the more items in the system, the better. This is a reasonable condition; our method applies to practical problems. In most industrial situations, e.g. that of a car-assemble plant, a very large number of auto parts are purchased from a relatively small number of suppliers, with each vendor supplying multiple parts.

6. Conclusion

This research addresses a multi-item joint replenishment problem in a stochastic setting. The integrated system we consider incorporates more realistic features on inventory control and transportation management. Our model provides a clear description of how these two issues affect each other, and of how to achieve a near-optimal inventory policy and vehicle routing schedule at the same time.
Our heuristic decomposition method employs a property of the combined problem to separate the model into subproblems, namely conventional inventory and vehicle routing models, each of which is solved with methods from existing literature. The inventory problem is dealt with item by item; for transportation, an optimal route is designed for each period. As a result, this method can be used to solve large-scale problems. We constructed a lower bound to the model, which was important in testing the effectiveness of the heuristic. A better lower bound was found for a special case when each supplier produces a unique item. Computational tests indicated that our method performed satisfactorily in solving the problem.

Appendix A

A.1. Notation and parameters

\[
\begin{align*}
P & \quad \text{total number of suppliers} \\
p & \quad \text{subscript denoting location (} p = 0, 1, \ldots, P; p = 0 \text{ for the central warehouse and } 1 \leq p \leq P \text{ for the suppliers)} \\
d_{op} & \quad \text{distance from warehouse to supplier } p \\
S & \quad \text{set of locations (warehouse and suppliers) which the vehicle may visit (} |S| = P + 1 \text{)} \\
k_{0p} & \quad \text{stopover cost at supplier } p \\
c & \quad \text{variable routing cost per unit distance travelled} \\
N & \quad \text{total number of items} \\
i & \quad \text{subscript denoting item (} i = 1, 2, \ldots, N \text{)} \\
\theta_i & \quad \theta_i = 1 \text{ if item } i \text{ is produced by supplier } p, \text{ and zero otherwise} \\
h_i & \quad \text{unit holding cost for item } i \text{ per unit time} \\
\pi_i & \quad \text{penalty cost per unit backlogged of item } i \\
K & \quad \text{joint fixed cost of ordering (including dispatching). } K \text{ is incurred if any of the } N \text{ items is ordered in a given period.} \\
k_{ri} & \quad \text{minor ordering cost for replenishing item } i \\
L & \quad \text{constant order lead time} \\
x_i & \quad \text{demand for item } i \text{ (} x_i \text{ is a random variable)} \\
\lambda_i & \quad \text{demand rate for item } i \\
\delta_i & \quad \text{diffusion coefficient of demand for item } i \\
j & \quad \text{subscript denoting period (} j = 1, 2, \ldots, M \text{). } M \text{ is calculated below.}
\end{align*}
\]

A.2. Decision variables and quantities calculated

\[
\begin{align*}
S_j & \quad \text{subset of } S, \text{ the locations (including warehouse) visited in period } j \text{ (} S_j \subseteq S \text{)} \\
D(S_j) & \quad \text{shortest travel distance for tour including the set of suppliers } S_j \text{ in period } j. \\
T & \quad \text{base period, when at least one item is ordered} \\
T_i & \quad \text{replenishment cycle for item } i \text{ (} T_i \text{ is an integer multiple of } T, \text{ i.e. } T_i = m_i T \text{)} \\
m_i & \quad m_i = T_i / T. \text{ (Item } i \text{ is ordered once in every } m_i \text{ base periods.)} \\
M & \quad \text{least common multiple of the } m_i \text{ (} MT \text{ is the total length of a regeneration cycle)} \\
R_i & \quad \text{order-up-to level for replenishment of item } i \\
y_{ij} & \quad y_{ij} = 1 \text{ if item } i \text{ is replenished in period } j, \text{ and zero otherwise} \\
\gamma_{pj} & \quad \gamma_{pj} = 1 \text{ if supplier } p \text{ is visited in period } j, \text{ and zero otherwise}
\end{align*}
\]
Appendix B

In this appendix, we present the allocation scheme for sharing transportation cost among items.

B.1. Travel cost

First, the travel cost \( cD(S_j) \) is allocated to suppliers based on respective distances from the central warehouse. The travel cost shared by supplier \( p \) is \( \left( d_{op} / \sum_{p=1}^{P} \gamma_{ip} d_{op} \right) cD(S_j) \). Then, allocation to each item collected from that supplier is on an equal basis. The share for item \( i \) is thus \( \left( y_{ij} \theta_{ip} / \sum_{i=1}^{N} \gamma_{ij} \theta_{ip} \right) \left( d_{op} / \sum_{p=1}^{P} \gamma_{ip} d_{op} \right) cD(S_j) \). Taking account (through \( \theta_{ip} \)) that the given item may be produced by several suppliers, we find the total travel cost for item \( i \) in period \( j \) as \( \sum_{p=1}^{P} \left( \left( y_{ij} \theta_{ip} / \sum_{i=1}^{N} \gamma_{ij} \theta_{ip} \right) \left( d_{op} / \sum_{p=1}^{P} \gamma_{ip} d_{op} \right) cD(S_j) \right) \).

B.2. Stopover cost

Stopover cost at a particular supplier is also apportioned equally among items replenished from that location in the given period. For item \( i \), its share of stopover cost at supplier \( p \) is \( \left( y_{ij} \theta_{ip} / \sum_{i=1}^{N} \gamma_{ij} \theta_{ip} \right) k_{tp} \). Accounting for all suppliers in period \( j \), the allocation of stop-over cost to item \( i \) is \( \sum_{p=1}^{P} \left( \left( y_{ij} \theta_{ip} / \sum_{i=1}^{N} \gamma_{ij} \theta_{ip} \right) k_{tp} \right) \). The sum over \( M \) periods of the above two costs gives the portion of total transportation costs incurred by item \( i \) over one entire repetition of the process. Since that item has been replenished \( M m_i \) times, its shared transportation cost per replenishment cycle is

\[
F_i = \left( m_i / M \right) \sum_{j=1}^{M} \sum_{p=1}^{P} \left( \left( y_{ij} \theta_{ip} / \sum_{i=1}^{N} \gamma_{ij} \theta_{ip} \right) \left( d_{op} / \sum_{p=1}^{P} \gamma_{ip} d_{op} \right) cD(S_j) + k_{tp} \right) \}
\]  

(12)

References


